ELECTRICITY AND MAGNETISM

Unit I : Electrostatics

Coulomb's law – electric field – electric dipole – electric flux – Gauss' s Law – applications – electric potential – relation connecting electric potential and electric potential at a point – potential at a point due to a point charge – potential due to an electric dipole – capacity – capacitance of a spherical and cylindrical capacitor – energy of a charged capacitor.

Unit II : Chemical Effects of Electric Current

Faraday' s Laws of Electrolysis – electrical conductivity of an electrolyte – specific conductivity – Kohlrausch bridge – Thermoelectricity – Seebeck effect – Peltier effect – Thomson effect – total e.m.f – thermodynamics of thermocouple – thermoelectric power diagram – its uses – applications.

Unit III : Transient Current

Growth and decay of current in a circuit containing resistance and inductance – Growth and decay of charge in a circuit containing resistance and capacitance – Determination of high resistance by leakage – Growth and decay of charge in a LCR circuit-

Unit IV : Alternating Current

I operator – properties – use of I operator in the study of A.C. circuit with R only – inductance only – capacitance only – LCR series and parallel circuits – power in an AC circuit – Wattless current – choke coil – construction and working of AC generator, 2 phase and 3 phase AC generator – distribution of 3 phase AC –

Unit V : Magnetic Properties of Materials

Magnetic induction – Magnetism – Relation between B, H and M – Magnetic susceptibility – Magnetic permeability – Relation between them – Electron theory of dia, para and ferromagnetism – Determination of susceptibility – Curie balance method – Moving coil Ballistic galvanometer – construction – theory – correction for damping in B.G – Measurement of Charge sensitiveness – absolute capacity of a condenser.

Books for Study and reference

1. Electricity and Magnetism - D.N.Vasudeva
2. Electricity and Magnetism - Brijlala and Subramanian
3. Electricity and Magnetism - R. Murugeshan
4. Electricity and Magnetism - K.K. Tewari

Unit - I

Electrostatics

1.1 Basic concepts :-

Electrostatics :- It deals with the behaviour of stationary charges. There are two kinds of electric charges : Positive and negative. Like charges repel each other and unlike charges attract each other. All charges in nature occur in integral multiples of the basic unit, ie., q = ne, where n is either a + ve or –ve integer. That is, the charge exists in discrete packets rather than in continuous amounts. That is, the charge is quantized.

From the law of conservation of electric charge, charge can neither be created nor destroyed. From the electrostatic behaviour, the materials are divided into two categories: conductors of electricity and insulators (dielectrics). Bodies which allow the charge or electricity to pass through them are called conductors, e.g.:- metals, human body, earth, graphite etc., Bodies which do not allow the charge or electricity to pass through them are called insulators. e.g:- glass, mica, ebonite, plastics.

1.2 Coulomb's Law :-

Statement :- The force between two point charges is directly proportional to the product of the charges and inversely proportional to square of the distance between them.

ie $F \propto q_1 q_2$

$\propto 1/r^2$

Where q_1 and q_2 are two point charges and r be the distance between the two charges.

$$\therefore \quad F = c \frac{q_1 q_2}{r^2}$$

Where c is a constant. In SI units $c = \frac{1}{4\pi\varepsilon_0}$

Where ε_O is called the permittirity of free space (ie vacuum).

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_2} \qquad (1.1)$$

The measured value of ε_o is 8.85418 x 10^{-12} C^{-12} N^{-1} m^{-12}, (or F M^{-1})

This gives, $\frac{1}{4\pi\varepsilon_0} = 9x10^9$ N m^2 C^{-2}

$\therefore$ Coulomb's law can also be written as,

$$F = \frac{q_1 q_2}{r^2} \text{ x } 9x10^9 \text{ Newtons}$$

For medium, the Coulomb's law may be written as,

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \qquad (1.2)$$

Where ε is the permittiviry of medium.

The relative permittivity ε_r medium

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$$

The value of ε_r for air is 1.

In equation (1.1), if $q_1 = q_2 = 1$ and $r = 1$, we have,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = 9\text{x}10^9 \text{ x} \frac{1\text{x}1}{1^2} = 9\text{x}10^9 \text{ Newtons}$$

The SI unit of charge is Coulomb.

A Coulomb is defined as the quantity of charge which, when at a distance of I metre in vacuum or air from an equal and similar charge experiences a repulsive force of $9\text{x}10^9$ N.

1.3 Electric Field :

Electric field at a point is defined as the force that acts on a unit +ve charge placed at that point.

$$E = \frac{F}{q}$$

Where F is the electrostatic force and q is the + ve electric charge.

The SI unit for electric field is Newton / Coulomb. For discrete stationary charges, the net electric field at a point is

$$E = E1 + E2 + E3 + \ldots\ldots\ldots\ldots \Sigma E_i$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + \ldots\ldots\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_i^2}{r_i^2}\right] \quad \text{Where } i = 1,2,3\ldots\ldots\ldots\ldots$$

For a continuous charge distribution, the electric field E at any point is given by

$$E = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}$$

1.4 Electric dipole :

Consider two charges –q at point A and +q at point B, the distance between them being 2d (Fig.1.1) such a charge configuration is called an electric dipole.

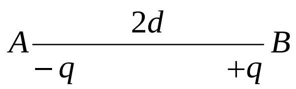

Fig. 1.1

The magnitude of the dipole moment is given by the product of any one of the charges and the distance between them

$$p = q \times 2d$$

The unit of p is Coulomb – metre.

1.5: Electric flux

The total number of lines of force cutting through a surface is called the electric flux through the surface.

In other words, the net outward flow or flux is the average out drawn normal components of the electric field E times the area of the surface. It is denoted by ϕ_E

$$\phi_E = (\text{average normal component of } E) \times \text{area}$$

The electric flux through a small area ds is shown in Fig.1.2.

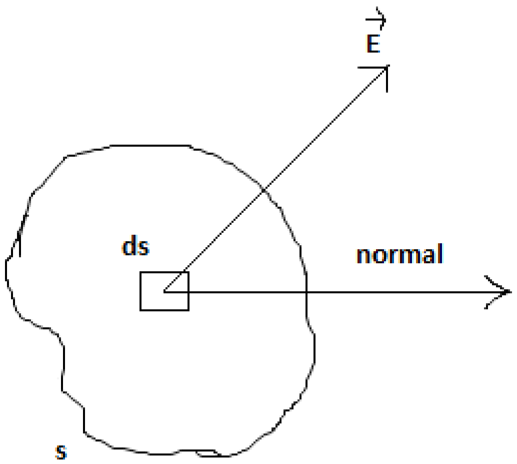

Fig.1.2

$$d\phi_E = E.\overrightarrow{ds} = E\,ds\,\cos\theta$$

Where θ is the angle between E and the normal to the area ds

The electric flux through the entire surface is

$$\phi_E = \int E.ds$$

If E is uniform over the entire surface area, we can write.

$$\phi_E = E..\vec{S}$$

The flux of the electric field is scalar. Its unit is N m^2 C^{-1} or Vm.

1.6: Gauss's Law :-

Statement :

The total flux of the electric field E over any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the total net charge enclosed by the surface.

$$\phi \quad = \quad \oint E.d\vec{s} = \frac{q}{\varepsilon_o}$$

Proof :-

(i) For a charge inside the closed surface

Consider a single point charge +q located at a point O inside a closed surface S (Fig.1.3). Let ds be a small area element at a distance r from q.

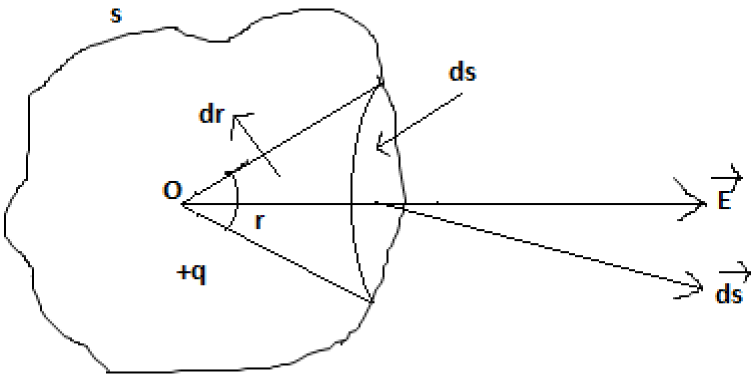

Fig.1.3

The electric field

$$E \quad = \quad \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

The flux through the area ds is given by

$$d\phi \quad = \quad E.\,ds \quad = Eds\,Cos\,\theta$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right) ds\,Cos\,\theta \quad = \quad \frac{q}{4\pi\varepsilon_o}\left(\frac{ds\,Cos\,\theta}{r^2}\right)$$

But $\frac{ds\,Cos\,\theta}{r^2} \quad = \quad d\Omega \quad = \quad$ Solid angle subtended by the area ds at O.

$$\therefore \; d\phi \quad = \quad \frac{q}{4\pi\varepsilon_o}d\Omega$$

$\therefore$ The total flux through the entire closed surface S is given by

$$\phi \quad = \quad \oint d\varphi \quad = \frac{q}{4\pi\varepsilon_o}\oint dr \quad = \frac{q}{4\pi\varepsilon_o} \text{ x } 4\pi \quad = \frac{q}{\varepsilon_o}$$

$\therefore\ \phi = \dfrac{q}{\varepsilon_o}$ Which is Gauss's law.

Gauss's law holds even if there are a number of charges q_1, q_2, ...q_n enclosed by a surface, S, because of the superposition principle.

(ii) For a charge outside the closed surface

Consider a point charge +q situated at 0 outside the closed surface (Fig.1.4)

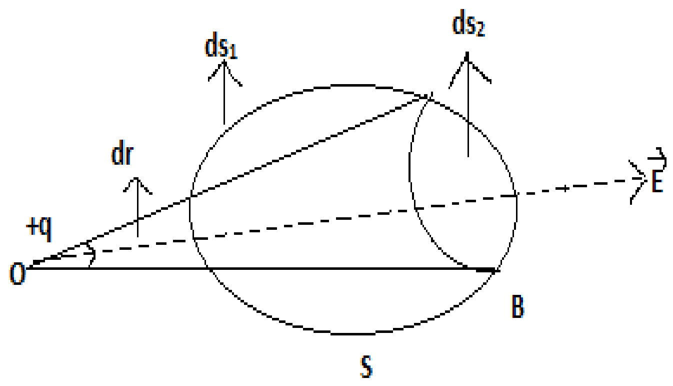

Fig.1.4

Let an elementary cone from 0 with small solid angle $d\Omega$ cut the closed surface at two elements of area ds_1 and ds_2. Magnitude of flux through ds_1 and ds_2 are equal. Therefore,

Total flux through ds_1 and ds_2 = $\dfrac{-q}{4\pi\varepsilon_0}\, d\Omega + \dfrac{q}{4\pi\varepsilon_0}\, d\Omega = 0.$

$\therefore$ The total flux due to a charge outside is Zero.

1.7: Differential from of Gauss's law :-

Suppose the charge is distributed over a volume. Let ρ be the charge density. Then the total charge within the closed surface enclosing the volume is given by

$$Q = \int \rho dV$$

We can write the integral form of Gauss's law as

$$\oint E.ds = \frac{1}{\varepsilon_o}\int \rho dV \qquad (1.3)$$

By Gauss divergence, theorem

$$\oint E.ds = \int (\nabla.E) dV \qquad (1.4)$$

Comparing the Eqs. (1.3) and (1.4), we get

$$\int (\nabla.E) dV = \frac{1}{\varepsilon_o}\int \rho dV$$

$$\therefore \quad \nabla . E = \frac{\rho}{\varepsilon_o}$$

This is differential form of Gauss's law.

1.8: Applications :

(1) Electric field due to a uniformly charged sphere :

A spherically symmetric charge distribution means the charge density ρ at any point depends only on the distance of the point from the centre and not on the direction.

Consider a total charge q distributed uniformly throughout a sphere of radius R.

Case (i) : When the point P lies outside the sphere

P is a point at a distance r from the centre O (Fig.1.5). Now we find the electric field E at P. Draw the concentric sphere (shown dotted) of radius OP with centre O. This is the Gaussian surface. At all points of this sphere, the magnitude of the electric field E is the same and its direction is perpendicular to the surface. Angle between E and ds is Zero.

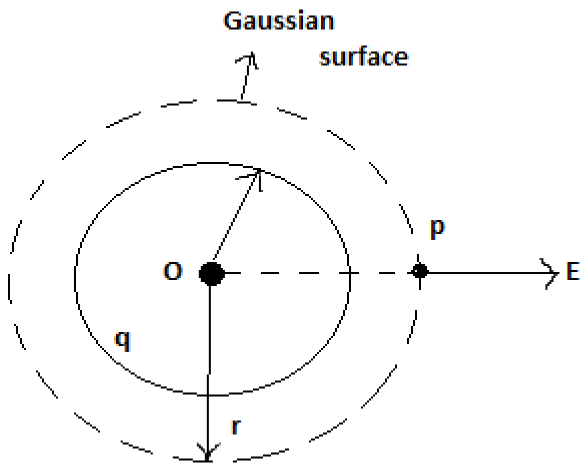

Fig.1.5

The flux through this surface is

$$\oint E.ds = \oint Eds = E \times 4\pi r^2.$$

By Gauss's law,

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_o}$$

$$\text{(or) } E = \frac{q}{4\pi\varepsilon_o r^2}$$

Hence the electric field at an external point due to uniformly charged sphere is the same as if the total charge is concentrated at its centre.

Case(ii) : When the point lies on the surface

Here r = R

$$\therefore E = \frac{q}{4\pi\varepsilon_o} \frac{q}{R^2}$$

Case(iii) : When the point lies inside the sphere

P^1 is a point inside the sphere (Fig 1.6). P^1 is at a distance r from the centre O. Draw a concentric sphere of radius r (r < R) in the centre at O. This is the Gaussian surface.

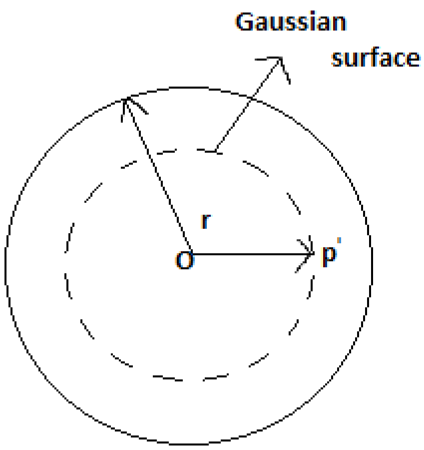

Fig.1.6

Total charge enclosed by the Gaussian surface

$$q^1 = \frac{4}{3}\pi r^3 \rho \times \frac{q}{\frac{4}{3}\pi R^3} = q\frac{r^3}{R^3}$$

Here P = charge density = Charge per unit volume = $\frac{q}{\frac{4}{3}\pi R^3}$

The outward flux through the surface of the sphere of radius r is

$$\oint E.ds = E \times 4\pi r^2$$

Applying Gauss's law,

$$E \times 4\pi r^2 = \frac{q^1}{\varepsilon_o} = \frac{q}{\varepsilon_o} \frac{r^3}{R^3}$$

$$\therefore E = \frac{q}{4\pi\varepsilon_o} \frac{r}{R^3}$$

Then $E \propto r$. At the centre of the sphere. E = 0.

2). Electric field due to an isolated uniformly charged conducting sphere (or) charged spherical shell.

In an isolated charged spherical conductor any excess charge on it is distributed uniformly over its surface and there is no charge inside it.

Case (i): At an external point :-

Consider a point P near but outside a uniformly charged sphere of radius R with a charge q (Fig.1.7). Let $\sigma = \frac{q}{4\pi R^2}$. P is at a distance r from the centre O. Draw a concentric sphere of radius OP with centre. This is the Gaussian surface.

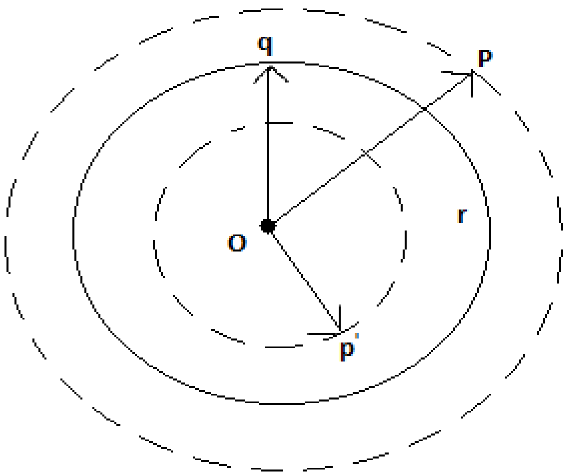

Fig.1.7

The flux through this surface is

$$\oint E.ds = \oint Eds = E \times 4\pi r^2.$$

By Gauss's law, $E \times 4\pi r^2 = \frac{q}{\varepsilon_o}$

(or) $$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

The E is therefore, the same as that due to a charge q situated at the centre of the sphere.

Case (ii) : At a point on the surface.

$$E = \frac{1}{4\pi\varepsilon_o} \frac{q}{R^2} \quad \text{(sincer r = R)}$$

Case (iii):- At a point inside :-

Let P[1] be an internal point. Through P[1] draw a concentric sphere. The charge inside this sphere is zero. Hence at all points inside the charged conducting sphere, E = 0.

3. Electric field due to a uniform infinite cyclndrical charge.

Let us consider that electric charge is distributed uniformly within an infinite cylinder of radius R. Let ρ be at charge density. Now we wish to find E at any point distant r from the axis lying (i) inside (ii) on the surface and (iii) outside the cylindrical charge distribution.

Case (i) :- When the point lies outside the charge distribution.

Let P_1 be a point at a distance r (>R) from the axis of the cylinder (Fig.1.8). Draw a coaxial cylinder of radius r and length l such that P_1 lies on the surface of this cylinder.

From symmetry, the Electric field E is every where normal to the curved surface and has the same magnitude all points on it. The electric flux due to plane faces is zero. So the total electric flux is due to the curved surface alone.

The electric flux due to curved surface $= \oint E.ds \quad = E \times 2\pi rl.$

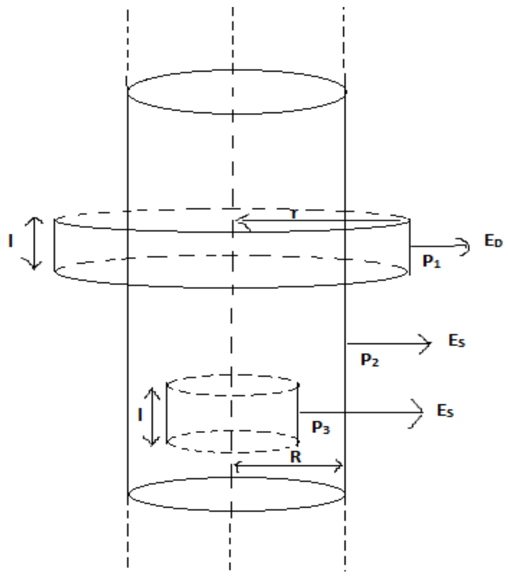

Fig.1.8

The net charge enclosed by the Gaussian surface = $q = (\pi R^2 l) \times \rho$

$\therefore$ By Gauss's law , $E \times 2\pi rl. = \pi R^2 l \rho / \varepsilon_o$.

(or) $E = \dfrac{\rho R^2}{2\varepsilon_o r}$

Case (ii) :- When the point lies on the surface of charge distribution (r = R)

Let P_2 be the point on the surface of charge distribution.

By Gauss's law,

$$E \times 2\pi Rl \quad = \quad \pi R^2 1 \rho / \varepsilon_0$$

$$\therefore E = \frac{\rho R}{2\varepsilon_o}$$

Case (iii):- When the point lies inside the charge distribution (r < R)

Let P_3 be the point at a distance (r < R) from the axis of the cylinder. Consider a coaxial cylindrical surface of radius r and length l such that P_3 lies on the curved surface of this cylinder.

The charge q^1 inside this Gaussian surface $= \pi R^2 l\rho$

By Gauss's law, $E \times 2\pi rl = \pi R^2 1\rho / \varepsilon_0$

$$E = \frac{r\rho}{2\varepsilon_o}$$

4. Field due to a uniformly charged Hollow cylinder

Consider a uniformly charged hollow cylinder of radius R (Fig.1.8) Let λ be the charge per unit length. P is a point at a distance r (r > R) from the axis of the cylinder. Draw a coaxial cylindrical Gaussian surface of radius r and length l. The electric flux due to the top and botton circular caps is zero.

The electric flux due to curved surface = $\oint E.ds =$ E x 2πrl. The net charge enclosed by the Gaussian surface = q = $\lambda 1$.

By Gauss's law, $E \times 2\pi rl = \frac{\lambda l}{\varepsilon_o}$

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

Let σ be the surface density of charge on the cylinder, then λ = 2πRσ

$$\therefore E = \frac{\sigma}{\varepsilon_o}$$

If we construct a Gaussian surface inside the hollow cylinder, it will encloses no charge. Therefore, the electric field inside a charged hollow cylinder is zero.

1.9: Electric potential

The electric potential at any point is defined as the work done in bringing a unit +ve charge from infinity to that point.

If W is the work done upon a charge 'q' to bring it from infinity to a given point in an electric field, then the potential at that point is given by

$$V = \frac{W}{q}$$

V is expressed in volts. It is a scalar quantity.

1.10: Potential difference

The workdone in moving a unit +ve charge between two points gives the potential difference between the two points (Fig.1.9).

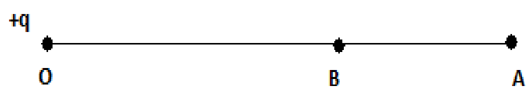

Fig.1.9

Mathematically.

$$V_A - V_B = \frac{W_{AB}}{q}$$

Where V_A and V_B stand and for the potential at A and B.

Electric potential in vector form :-

Let A and B be two points in a non uniform electric field. Let a test charge 'q' move from A to B along any path (Fig.1.10).

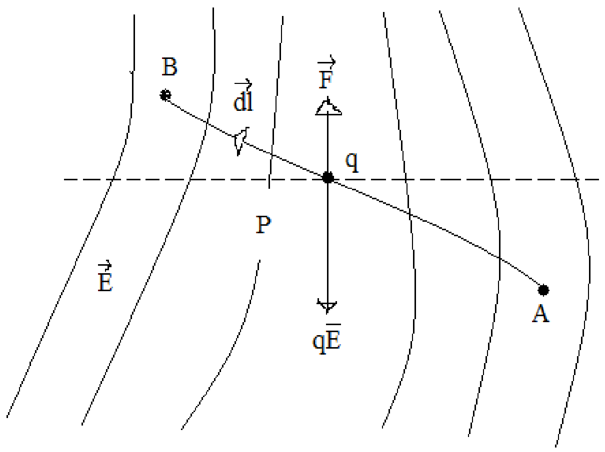

Fig.1.10

Let E be the electric field at any point P. The electric field exerts a force qE on the charge q. That is

$$F = -qE$$

The work done for a small displacement dl along AB = F.dl

$\therefore$ The total work done in moving the charge from A to B is

$$W_{AB} = \int_A^B F.dl = -q\int_A^B E.dl \qquad (\because F = -qE)$$

(or) $\frac{W_{AB}}{q} = -\int_A^B E.dl$

But $V_A - V_B = \frac{W_{AB}}{q}$

$$\therefore V_A - V_B = -\int_A^B E.dl$$

It the point A lies at infinity. V_A = o, Then the potential at the point B is

$$V_B = -\int_A^B E.dl$$

1.12: Relation between the electric field and electric potential

The potential difference between two points in an electric field depends only on the coordinates of those points and is independent of the path taken in going from one point to the other.

To find the electric field in terms of electric potential, consider the potential at two neighboring points *A(x,y,z)* and B *(x+dx, y+ dy, z+dz)* at a distance 'dl' apart in the region. The potential difference V is going from A to B is given by

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz \qquad (1.5)$$

But we know

$$dv = - E.\, dl \qquad (1.6)$$

Comparing (1.5) and (1.6), we get

$$-E.\, dl = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz$$

Also $l = x + iy + zk$

$$\therefore dl = i\, dx + j\, dy + k\, dz$$

$$\therefore -E.\, \overrightarrow{dl} = -E.\,(i\, dx + j\, dy + k\, dz) = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz$$

$$= \left(i\frac{\partial v}{\partial x} + j\frac{\partial v}{\partial y} + k\frac{\partial v}{\partial z}\right).\,(i\, dx + j\, dy + k\, dz)$$

$$\therefore -E = \left(i\frac{\partial v}{\partial x} + j\frac{\partial v}{\partial y} + k\frac{\partial v}{\partial z}\right)$$

$$-E = \text{grad } v = \nabla v$$

(or) E = - ∇v where $\nabla = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right)$

This is the relation connects the electric field and electric potential.

1.13: Potential due to a point charge :-

Let +q be an isolated point – charge situated in air. P is a point distant r from +q (Fig.1.11)

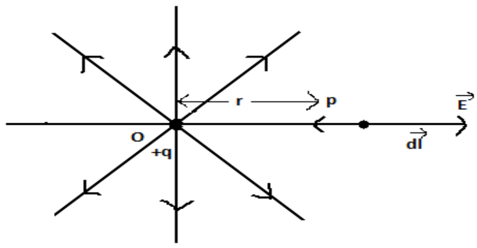

Fig.1.11

∴ The electric filed $E = \frac{1}{4\pi\varepsilon_o} \frac{q}{r^2}$ (1.7)

The potential at P is given by

$$V = \int_{\infty}^{r} E.dl \qquad (1.8)$$

The displacement dl of the unit charge is directed towards the left. E is directed towards the right. Thus the angle between E and dl is 180.

∴ E.dl = Edl Cos 180° = -E dl.

r is measured from the charge +q as origin. As we move a distance dl to the left, the value of r decreases. Thus dl = -dr.

∴ E.dl = -E dl = E dr.

Thus equation (1.8) becomes,

$$V = \int_{\infty}^{r} E.dl = V = \int_{\infty}^{r} E.dl = -\frac{q}{4\pi\varepsilon_O} \int_{\infty}^{r} \frac{dr}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_O} \frac{q}{r}$$

This is the expression for the potential at a point r due to a point charge.

1.14. The electric potential at a point due to a dipole.

Consider a dipole AB with +q charge at A and –q charge at B separated by a distance $2l$ as shown in Fig.1.12. Let 0 be the mid point of the dipole. Let P be a point in free space at a distance r from 0 and let angle ∠POB = θ, Ap = r_1 and BP = r_2.

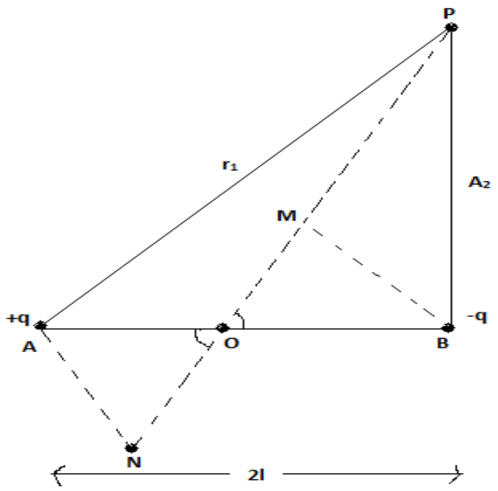

Fig.1.12

The electric potential at P due to the charge +q $= \dfrac{q}{4\pi\varepsilon_o r_1}$.

The electric potential at P due to thecharge –q $= \dfrac{-q}{4\pi\varepsilon_o r_2}$

Hence the net potential at P due to the dipole is

$$V = \frac{q}{4\pi\varepsilon_o r_1} - \frac{-q}{4\pi\varepsilon_o r_2}$$

$$\text{(or)} \quad V = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \qquad (1.9)$$

Let MB and AN be drawn perpendicular to PO. Then OM = ON = l cos θ. As r >>l, BP≈ MP = r-OM = r – l Cos θ and AP = PN = r + ON = r + l cos θ.

Hence from Eq. (1.9), we have

$$V = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\frac{r - l\cos\theta - r + l\cos\theta}{(r^2 - l^2\cos^2\theta)}$$

$$= \frac{2lq\cos\theta}{4\pi\varepsilon_o(r^2 - l^2\cos^2\theta)}$$

Since r >>l, the quantity $l^2 \cos^2 \theta$ can be neglected.

The product 2lq in called electric dipole moment

$\therefore$ 2lq = P

Hence $V = \frac{p\cos\theta}{4\pi\varepsilon_o r^2}$

Special cases:-

(i) When point P lies on the axial line of the dipole on the side of the +ve charge q, θ=0, Cos θ = 1.

$$\therefore \quad V = \frac{p}{4\pi\varepsilon_o r^2}$$

(ii) When the point P lies on the axial line of the dipole on the side of the –ve charge q, θ=180°, Cos θ = -1.

$$\therefore V = \frac{P}{4\pi\varepsilon_o r^2}$$

(iii) When the point P lies on the equatorial line of the dipole.

$$\theta = \pi/2, Cos\,\pi/2 = 0, \therefore V = 0.$$

1.15 Capacitance of a conductor

The ratio Q/V is called the capacitance of the conductor and is denoted by C.

C = Q/V

The unit of capacitance is *Farad.*

A conductor has a capacitance of one Farad, if a charge of 1 coulomb given to it raises its potential by 1 volt.

1μF = 10^{-6}F, 1pF = 10^{-12}F.

1.16: Capacitance of a spherical capacitors :-

(i) Outer sphere earth connected :

A and B are two spherical conductors of radii a and b. A is charged and B is earthed as shown in fig.1.13. Let the charge on the conductor A be +q. Due to induction, a charge –q is induced on the inner substance of the outer sphere. A uniform electric field E is established between the spheres directed from A to B radially. Consider the element of radius x and thickness dx. Then electric field at P_1,

$$E = \frac{q}{4\pi\varepsilon_o x^2}$$

Potential difference between P_1 and P_2 = dv

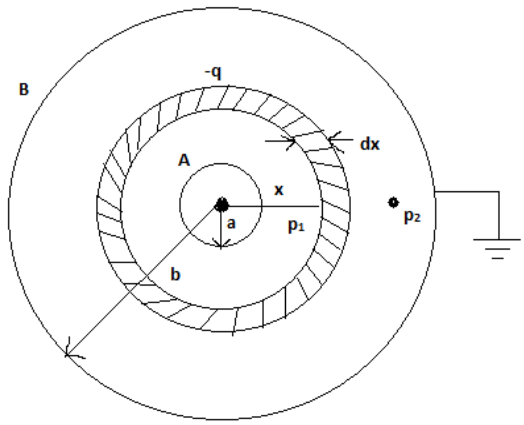

Fig.1.13

$$\therefore \quad dv = -E\,dx = -\frac{q}{4\pi\varepsilon_o x^2}dx$$

Potential difference between A and B,

$$V = \int_b^a -\frac{q}{4\pi\varepsilon_o x^2}\,dx$$

$$= -\frac{q}{4\pi\varepsilon_o x^2}\int_b^a \frac{dx}{x^2} = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{x}\right]_b^a$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{a}-\frac{1}{b}\right] = \frac{q}{4\pi\varepsilon_o}\left[\frac{b-a}{ab}\right]$$

$$\therefore \quad V = \frac{q}{4\pi\varepsilon_o}\left[\frac{b-a}{ab}\right]$$

But $\quad C = q / V = \dfrac{q}{\frac{q}{4\pi\varepsilon_o}\left[\frac{b-a}{ab}\right]} = \left[\dfrac{4\pi\varepsilon_o ab}{b-a}\right]$

$\therefore$ The capacity of the spherical capacitor

$$C = \left[\frac{4\pi\varepsilon_o ab}{b-a}\right]$$

For Medium

$$C = \left[\frac{4\pi\varepsilon_O \ \ \varepsilon_r \ ab}{b-a}\right]$$

ii) Inner sphere earth connected :-

A Charge +q is given to the outer conductor, $+q_1$ is on its inner surface and $+q_2$ is on its outer surface as shown in fig.1.14.

$q = q_1 + q_2$

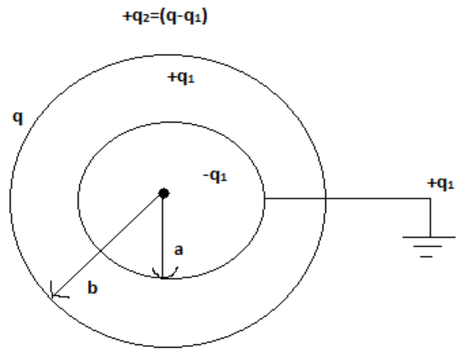

Fig.1.14

The + q_1 charge on the inner surface of the outer conductor induces charge - q_1 on the inner sphere and + q_1, flows to the earth. The charge $+q_1$ on the inner surface of the outer sphere and - q_1 on the inner sphere form a spherical capacitor whose capacitance will be

$$= \frac{4\pi\varepsilon_O ab}{b-a}$$

The capacitance of the outer surface of the sphere having a charge $+q_2$ and radius b = $4\pi\varepsilon_o b$.

$$\therefore \text{Total capacitance } C = \frac{4\pi\varepsilon_O ab}{b-a} + 4\pi\varepsilon_O ab$$

$$= 4\pi\varepsilon_O \left[\frac{ab}{b-a} + b\right] = 4\pi\varepsilon_O \left[\frac{ab + b^2 - ab}{b-a}\right]$$

$$= \frac{4\pi\varepsilon_o ab^2}{b-a}$$

$\therefore$ Capacitance of the spherical capacitor with its inner sphere earthed in the air medium

$$C = \frac{4\pi\varepsilon_O ab}{b-a}$$

(iv) **Capacitance of isolated conducting sphere :-**

The capacitance of the spherical capacitor is

$$C = \frac{4\pi\varepsilon_O ab}{b-a} = \frac{4\pi\varepsilon_O}{\frac{1}{a} - \frac{1}{\infty}}$$

If the outer sphere is infinite radius (b=∞) then we are left with an isolated conducting sphere. Its capacitance is (put b =∞) in the above equation.

$$\therefore \quad C = \frac{4\pi\varepsilon_O}{\frac{1}{a} - \frac{1}{\infty}} = 4\pi\varepsilon_O a$$

Then the capacitance of an isolated sphere of radius ‘a’ metre is $4\pi\varepsilon_o a$ Farad.

1.17: Capacitance of a cylindrical capacitor.

A cylindrical capacitor consists of two coaxial cylinders of radius a and length *l* as shown in fig.1.15. We assume that *l* >> b, ie the length of the cyclinder *l* is large compared to the radius of the cylinder. Let the inner cylinder A be given a charge +q and the outer cylinder B be earthed. Due to the electrostatic induction the inner surface of B is charged to –q. To find the electric field intensity E between the cylinders, consider a Gaussian cylindrical surface of radius r and length *l* closed by plane caps as shown in fig.1.15(a).

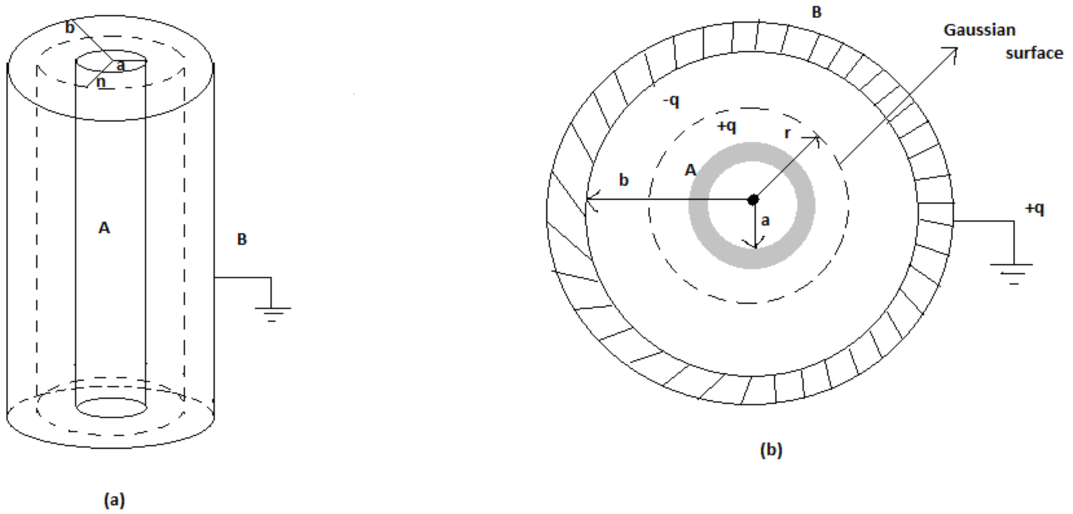

Fig.1.15

According to Gaussis law,

$$\varphi = \int E.ds = \frac{q}{\varepsilon_O}$$

The flux is normal to the Gaussian cylindrical surface and no flux passes through the plane caps. Therefore the above surface integral involves only the cyclindrical surface.

$$\text{ie} \qquad E\oint ds = \frac{q}{\varepsilon_O}$$

$$E \times 2\pi r l = \frac{q}{\varepsilon_O}$$

$$\therefore E = \frac{q}{2\pi\varepsilon_O rl}$$

The potential difference between the cycllinders is given by

$$V = -\int_b^a \vec{E}.\vec{dr}$$

As the angle between E and dr is zero, since E is radial ie along $\vec{r}$. So, $\vec{E}.\vec{dr}$ = Edr. Hence

$$V = -\int_b^a Edr = -\int_b^a \frac{q}{2\pi\varepsilon_O rl} dr$$

$$V = \frac{q}{2\pi\varepsilon_O rl} \log_e \left(\frac{b}{a}\right)$$

$$\therefore \text{ The capacitance } C = \frac{q}{V} = \frac{q}{\frac{q}{2\pi\varepsilon_O rl}\log_e(\frac{b}{a})}$$

$$C = \frac{2\pi\varepsilon_O rl}{\log_e\left(b/a\right)}$$

$\therefore$The capacitance of the cylindrical capacitor is

$$C = \frac{2\pi\varepsilon_O rl}{2.3026 \times \log_{10}\left(b/a\right)}$$

It the space between the cylinders is filled with a dielectric of dielectric constant ε_r, then,

$$C = \frac{2\pi\varepsilon_O \varepsilon_r l}{2.3026 \times \log_{10}\left(b/a\right)}$$

1.18 Energy stored in a capacitor

The energy of a charged condenser is equal to the workdone in charging it. The workdone (dW) in bringing a small charge dq to the capacitor when the potential is V, then

dW = Vdq

Hence the total workdone in charging it with a charge q is

$$W = \int_b^a Vdq$$

We know

$$V = {}^{q}\!/_{c}, \qquad \therefore W = \int_0^q \frac{q}{c}\,dq = \frac{1}{C}\int_o^q q\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$$

$$W = \frac{1}{2}\frac{q^2}{C}$$

But $q = CV$

$$\therefore \text{ Workdone} = \frac{1}{2}\frac{c^2V^2}{C} = \frac{1}{2}CV^2$$

$$\therefore W = \frac{1}{2}CV^2 \text{ Joule}$$

This is the work done to store the energy in a charged condenser.

1.19: Loss of energy during sharing of charges :

When two conductors at different potential are connected together by a wire, the charge flows, from the conductor at the higher potential to that at the lower potential, until their potentials are equal. During this process the system loses energy.

Let A and B be the two conductors of capacities C_1 and C_2 charged to potentials V_1 and V_2 respectively as shown in fig.1.16.

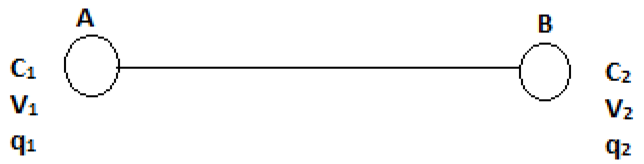

Fig:1.16. Sharing of charges between two charged conductors.

Energy stored in the conductor A before contact = $\frac{1}{2}c_1V_1^2$

Energy stored in the conductor B before contact = $\frac{1}{2}c_2V_2^2$

Total energy stored before contact (E_1) = $\frac{1}{2}c_1V_1^2 + \frac{1}{2}c_2V_2^2$

when the two conductors are joined by a wire, the common potential

$$V = \text{Total charge / total capacity}$$

$$= \frac{c_1V_2 + c_2V_2}{c_1 + c_2}$$

Total energy of the conductors after contact

$$E_2 = \frac{1}{2}(c_1 + c_2)V^2$$

$$\therefore E_2 \quad = \quad \frac{1}{2}(c_1+c_2)\ \frac{(c_1V_1+c_2V_2)^2}{(c_1+c_2)^2}$$

$$= \quad \frac{(c_1V_1+c_2V_2)^2}{2(c_1+c_2)}$$

Loss of energy due to contact,

$$E_1 - E_2 \quad = \quad \frac{1}{2}c_1V_1^2 + \frac{1}{2}c_2V_2^2 - \frac{(c_1V_1+c_2V_2)^2}{(c_1+c_2)^2}$$

$$= \quad \frac{1}{2(c_1+c_2)^2}\left[(c_1+c_2)(c_1V_1^2+c_2V_2^2)-(c_1V_1+c_2V_2)^2\right]$$

$$= \quad \frac{1}{2(c_1+c_2)}\left[(c_1+c_2)(V_1^2+V_2^2)-(2c_1c_2+V_1V_2)\right]$$

$$= \quad \frac{c_1+c_2}{2(c_1+c_2)}\left[(V_1^2+V_2^2)-2V_1V_2)\right]$$

$$= \quad \frac{c_1+c_2}{2(c_1+c_2)}\left[(V_1+V_2)^2)\right]$$

$$E_1 - E_2 \quad = \quad \frac{c_1c_2}{2(c_1+c_2)}\left[V_1-V_2\right]^2$$

Since $(V_1 - V_2)^2$ in always +ve quantity, E_2 must be less than E_1. Hence there is a loss of energy on sharing their charges. The balance of energy (E_1-E_2) appears partly as heat in the connecting wires and partly as light and sound if sparking occurs.

Unit – II

Chemical Effects of Electrical Current

2.1 Electrolysis :-

When an electric current passes through certain compounds in solution or molten state, they are decomposed. The decomposition of compounds by an electric current is called *electrolysis.*

These compounds are known as ***electrolytes*.** Acids, bases and metallic salts are some examples of ***electrolytes*.** Alcohol and glycerine are some examples of ***non-electrolytes*.**

Examples :

1. When an electric current is passed through the dilute sulphuric acid (H_2So_4 using platinum electrodes, the H_2So_4 is decomposed into hydrogen and sulphate radical.

$$H_2So_4 \xrightarrow[current]{electric} H_2 + So_4$$

2. During the electrolysis of sodium chloride using platinum electrodes, it decomposes as follows.

$$NaCl \rightarrow Na + Cl\uparrow$$

2.2 Faraday's law of electrolysis

(1) First law :

The mass of an element or ion liberated (or deposited) from an electrolyte at the respective electrodes is directly proportional to the quantity of electricity.

$$m \,\alpha\, q \qquad \text{but} \qquad q = it$$

$$m \,\alpha\, it$$

(or) $$\boxed{m = z\, it} \qquad \text{or} \qquad \boxed{m = zq}$$

Where *z* is a constant called electro chemical equivalent. (e.c.e)

$$\therefore z = \frac{m}{q} = \frac{\text{grams}}{\text{coulomb}}$$

II. Second law :-

If the same quantity of electricity passes through different electrolytes, the masses of the elements (or ions) liberated at the respective electrodes are proportional to their chemical equivalent.

ie $$\frac{m_1}{E_1} = \frac{m_2}{E_2} = \frac{m_3}{E_3} = \text{a constant}$$

(or) $$m_1 = z_1 q,\ m_2 = z_2 q,\ m_3 = z_3 q$$

$$\therefore \frac{z_1 q}{E_1} = \frac{z_2 q}{E_2} = \frac{z_3 q}{E_3} = \text{a constant} = 1\,\text{Faraday}$$

(or) $$\frac{z_1}{E_1} = \frac{z_2}{E_2} = \frac{z_3}{E_3} = \text{a constant} = 1\,\text{Faraday}$$

1Faraday :-

It may be defined like this, one faraday is the quantity of electricity that must be passed through an electrolyte to liberate one gram equivalent of a element.

(or) It may also be define like this, ie

1 Faraday = Avagadro's number x charge on the electron.

2.3: Electrical conductivity of an electrolyte

The current passing through an electrolyte is given by

$$I = \frac{E - e}{R}$$

where E is the applied *emf*, *e* is the back *emf* due to polarization and R is the resistance of the electrolyte. The variation of current with the applied *emf* is show in Fig.2.1.

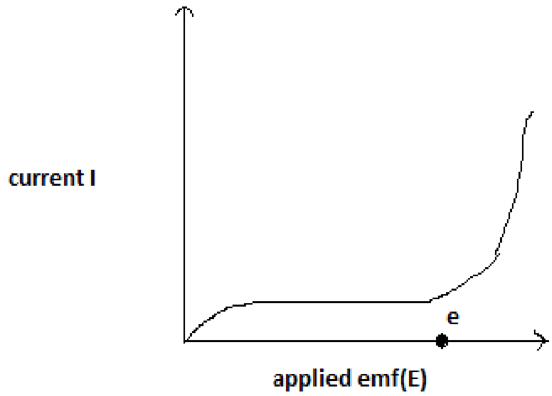

Fig.2.1

If *l* is the length of the electrolyte through which the current passes and *'a'* is area of cross - section of the electrodes, then

$$R \alpha \frac{l}{a} \text{ (or) } R = \rho \frac{l}{a}$$

Where ρ is the specific resistivity of the electrolyte

ie $$\rho = \frac{Ra}{l} \quad \text{ohm – metre}$$

The reciprocal of the specific resistivity is called specific conductivity (σ) of the electrolyte.

$$\sigma = \frac{1}{\rho} ohm^{-1} \quad m^{-1}$$

The equivalent conductivity (λ) of an electrolyte is defined as the ratio of the specific conductivity σ to the concentration C,

$$\lambda = \frac{\sigma}{C}$$

2.4: Determination of specific conductivity of electrolytes – Kohlrausch Bridge

The cell containing the electrolyte whose conductivity has to be determined is introduced into one arm (R) of a wheatstone's bridge as shown in Fig.2.2.

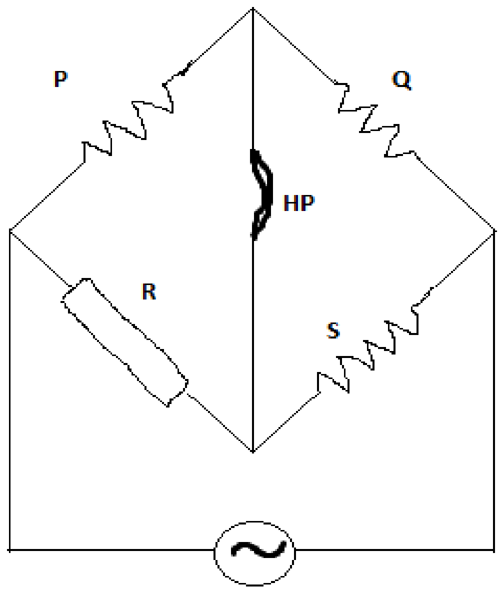

Fig.2.2

The other resistances are P,Q and S. To detect the balancing point of the bridge, a head – phone (HP) is connected when the state of balance is indicated by minimum sound through the head phone. An alternating current (a.c) is passed through the network and the resistances P,Q and R adjusted for minimum sound through the head phone. Then,

$$\frac{P}{Q} = \frac{R}{S}$$

or

$$R = \frac{P}{Q} \text{ x } S$$

If l is the length of column of electrolyte in the tube, a is the area of each plate, and ρ is the specific resistance

$$\therefore \; R = \frac{\rho l}{a} \quad \text{(or)} \quad \rho = \frac{Ra}{l}$$

The specific conductivity σ of the electrolyte is then determined by $\sigma = \frac{1}{\rho}$

2.5: Thermo electricity

It deals with heat energy is transformed into electrical energy or vice versa, ie., current produced without use of a cell or a battery and this current is known as thermoelectric current.

2.5.1: Seebeck effect

When two dissimilar metal wires are joined together so as to form a closed circuit and if the two junctions are maintained at different temperatures, an emf is developed in the circuit (Fig.2.3). This phenomenon is called the *Seebeck effect***.** This arrangement is called *thermocouple.* The emf developed is called *thermo emf***.** Seebeck arranged the metals in a series as

Bi, Ni, Pd, Pt, Cu, Mn, Hg, Pb, Sn, Au, Ag, Zn, Cd, Fe, Sb

when a thermocouple is formed between any two of them, the thermoelectric current flows through the hot junction from the metal occurring earlier to the metal occurring later in the list. The metals to the left of Pb are called thermoelectrically negative and those to its right are thermoelectrically positive.

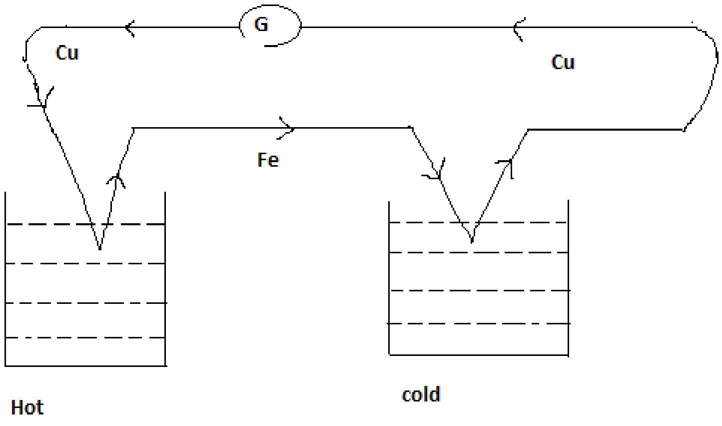

Fig.2.3

2.5.2: Variation of thermo *emf* with temperature

If the temperature of the cold junction of thermocouple be kept at O°C and the thermo *emf* 'e' plotted against the temperature T of the hot junction, the graph is a parabola as shown in Fig.2.4.

The temperature of the hot junction at which the thermo *emf* becomes maximum is called the *neutral temperature* (T_n) and is a constant for given pair of metals.

The temperature at which the reversal of thermo *emf* takes place is called the *temperature of inversion*.

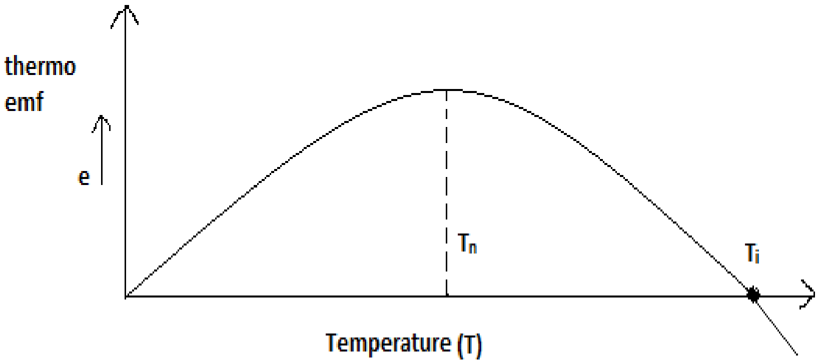

Fig.2.4

The relation between *emf* *'e'* and the temperature T is expressed by the equation,

$$e = aT + bT^2 \quad (2.1)$$

where *'a'* and *'b'* are constants.

Differentiating the Eq.(2.1) we get,

$$\frac{de}{dT} = a + 2bT$$

At $T = T_n$, e is maximum, ie $\frac{de}{dT} = 0.$

Thus, $0 = a + 2bT_n$

(or) $T_n = -a / 2b$ (2.2)

At $T = T_i$, $e = 0$

$\therefore \; 0 = a\,T_i + bT_i^2$

or $T_i = -a / b$ (2.3)

Comparing Eqs. (2.2) and (2.3), we get,

$$\boxed{T_i = 2T_n}$$

2.6: Laws thermo emf

1. Law of intermediate metals :-

This law states that the addition of a third metal into any thermoelectric current does not alter the thero *emf*, provided the metal is at the same temperature at the point where it is introduced.

If ${}_aE_b$ is the *emf* for a couple made of metal A and B and ${}_bE_c$ that for the couple of metals B and C, then the *emf* for couple of metals A and C is given by

$${}_aE_c = {}_aE_b + {}_bE_c$$

2. Law of intermediate temperatures :-

The thermo emf E_1^3 of a thermocouple whose junctions are maintained at temperatures T_1 and T_3 is equal to the sum of the emf E_1^2 and E_2^3 when the junctions are maintained at temperatures T_1, T_2 and T_2, T_3 respectively, Thus,

$$E_1^3 = E_1^2 + E_2^3$$

2.7: Peltier effect

When a current is passed through a circuit formed by two dissimilar metals, heat is evolved at one junction and absorbed at the other junction. This effect is known as *Peltier effect.* Peltier effect is a reversible effect.

The amount of heat H absorbed or evolved at a junction is proportional to the charge 'q' passing through the junction, ie.,

$$H \alpha q \quad \text{(or)} \quad H \alpha it$$

(or) $$H = \pi it$$

Where π is a constant called *Peltier coefficient*.

When I = 1 ampere and t= 1 sec then $H = \pi$

Ie., the energy that is liberated or absorbed at a junction between two dissimilar metals due to the passage of unit quantity of electricity is called *Peltier coefficient*.

Difference between Peltier and Joule's effect.

Peltier effect		Joule's effect	
1.	It is a reversible effect	1.	It is not a reversible effect
2.	It takes place at the junctions only	2.	It takes place throughout the conductor.
3.	It may be a cooling or heating effect	3.	It is always a heating effect
4.	It is directly proportional to I (ie., $H=\pi it$)	4.	It is directly proportional to square of the current ($H=I^2R$)
5.	It depends upon the direction of the current	5.	It is independent of the direction of the current

2.8: Thomson effect :-

When a current flows through an unequally heated metal, there is an absorption or evolution of heat energy absorbed or evolved when a charge of the 1 coulomb flows in the metal between two points which differ in temperature by 1°C.

The Thomson effect is reversible one.

The metals like Ag, Zn, Sb and Cd shows +ve Thomson effect.

The mtals like Pt, Ni, Co and Bi shows –ve Thomson effect.

For lead, the Thomson effect is zero.

Thomson coefficient (σ)

The Thomson coefficient σ of a metal is defined as the amount of heat energy absorbed or evolved when a charge of 1 coulomb flows in the metal between two points which differ in temperature by 1°C.

It is expressed in joules per coulomb per °C. (or) volt / °C.

2.9: Total *emf* in a thermocouple.

Let J_1, and J_2 be the hot and cold junction of thermocouple made of metals A and B. Let $T_1°$ and $T_2°$ be the absolute temperature of the hot and cold junctions and π_1 and π_2 the Peltier coefficients at these temperature. Let σ_A and σ_B the Thomson coefficient of the metal A and B both taken +ve for simplicity (Fig. 2.5).

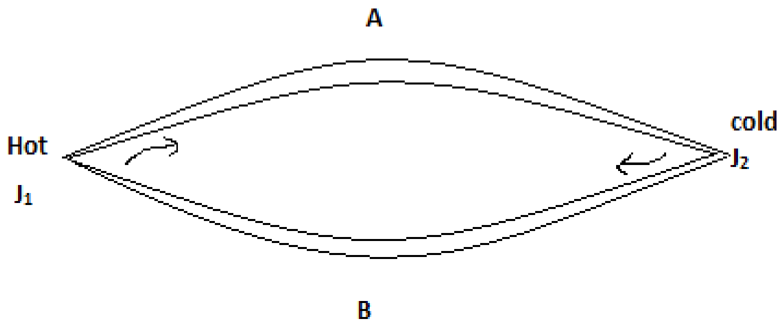

Fig.2.5

The net *emf* in volts acting in the circuits given by the workdone in taking 1 coulomb of charge completely round the circuit once.

Energy absorbed at J_1 due to Peltier effect = π_1 joules

Energy liberated at J_2 due to Peltier effect = π_2 joules

Energy liberated at A due to Thomson effect = $\int_{T_2}^{T_1} \sigma_A \, d_T$ joules

The energy absorbed at B due to Thomson effect

$$\int_{T_2}^{T_1} \sigma_B \, d_T \text{ joules}$$

Net energy absorbed

$$E = \pi_1 - \pi_2 - \int_{T_2}^{T_1} (\sigma_A - \sigma_B) dT$$

But this energy gives resultant *emf* in the thermocouple

ie

$$E = \pi_1 - \pi_2 - \int_{T_2}^{T_1} (\sigma_A - \sigma_B) dT$$

2.10: Thermo electric power :-

It is defined as the rate of change of thermo *emf* with respect to temperature

We know

$$E = aT + bT^2$$

$$\frac{dE}{dT} = a + 2bT \quad \text{which is called Thermoelectric power}$$

If a graph is plotted between thermoelectric power and temperature, it is a *straight line*.

2.11: Thermodynamics of Thermocouple

Let A and B be two metals forming of a thermocouple, with one junction at a lower temperature T and the other at a higher temperature T + dT as shown in Fig: 2.6.

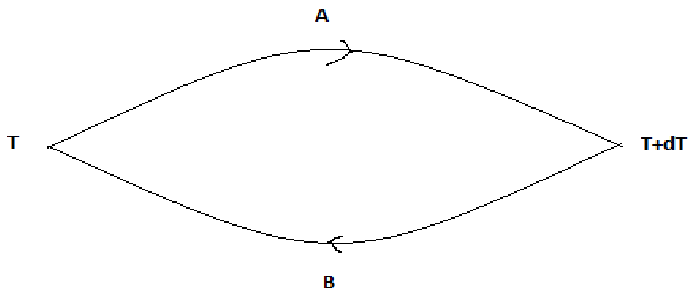

Fig.2.6

Let π and $\pi + d\pi$ be the Peltier coefficients at T and T + dT respectively. Let σ_A and σ_B be the Thomson coefficients for the metals A and B respectively. Then, assuming the thermoelectric current to pass from A to B at the hot junction, the energy gained by unit quantity of electricity is $\pi + d\pi$ and $-\pi$ due to the Peltier effect, $\sigma_A\, dT$ and $\sigma_B\, dT$ due to the Thomson effect. Hence the total gain of energy by unit quantity of electricity for the complete circuit is

$$\pi + d\pi - \pi + \sigma_A\, dT - \sigma_B\, dT = d\pi + (\sigma_A - \sigma_B)\, dT$$

Since this gain to energy is the numerically equal to the thermo *emf dE* in the circuit

$$dE = d\pi + (\sigma_A - \sigma_B)\, dT \qquad (2.4)$$

From IInd law of thermodynamics,

$$\frac{\pi + d\pi}{T + dT} - \frac{\pi}{T} + \frac{\sigma_A - \sigma_B}{T} dT = 0.$$

or $$T(\pi + d\pi) - \pi(T+dT) + (T+dT)(\sigma_A - \sigma_B)\, dT = 0$$

$$T d\pi - \pi dT + T(\sigma_A - \sigma_B)\, dT = 0.$$

(neglecting the term involving dT^2)

(or) $$d\pi - \frac{\pi}{T} dT + (\sigma_A - \sigma_B) dT = 0$$

(or) $$(\sigma_A - \sigma_B) dT = \frac{\pi}{T} dT - d\pi \quad (2.5)$$

Substituting the *emf* (Eq. 2.5) in Eq. (2.4) we get,

$$dE = d\pi + \frac{\pi}{T} dT - d\pi \quad (2.6)$$

(or) $$\pi = T \frac{dE}{dT} \quad (2.7)$$

Thus, the Peltier coefficient for a junction of a pair of metals is the product of the absolute temperature (T) of the junction and the thermoelectric power at the temperature.

Peltier coefficient = Absolute temp x Thermoelectric power

From Eq.(2.4), we have,

$$\sigma_A - \sigma_B = \frac{\pi}{T} - \frac{dE}{dT} \quad (2.8)$$

By differentiating the Eq. (2.7) we get

$$\frac{d\pi}{dT} = T \frac{d^2E}{dT^2} + \frac{dE}{dT} \quad (2.9)$$

Substituting the values from Eqs. (2.7) and (2.9) in Eq. (2.8) we get,

$$\sigma_A - \sigma_B = \frac{dE}{dT} - T\frac{d^2E}{dT^2} - \frac{dE}{dT}$$

Or $$\boxed{\sigma_A - \sigma_B = -T\frac{d^2E}{dT^2}}$$

If the metal A is lead, $\sigma_A = 0$, Hence

$$\boxed{\sigma_B = T\frac{d^2E}{dT^2}}$$

2.12: Thermo – Electric Power diagrams

Thermo – electric power diagram often called Tait's diagram is a plot of thermo – electric power P against the temperature.

We know

$$E = aT + bT^2$$

A graph between E and T is a parabola

$$\frac{dE}{dT} = a + 2bT$$

Where $\frac{dE}{dT}$ is called thermo – electric power.

A graph between thermo electric power (dE/dT) and difference of temperature (T) is a straight line (Fig.2.7). This graph is called Thermo – electric power line (or) the thermo electric diagram. Thomson coefficient of lead is zero. So generally thermoelectric lines are drawn with lead as one metal of the thermocouple. The thermocouple line of a Cu-Pb couple has a +ve slope while that of Fe - Pb couple has a -ve slope.

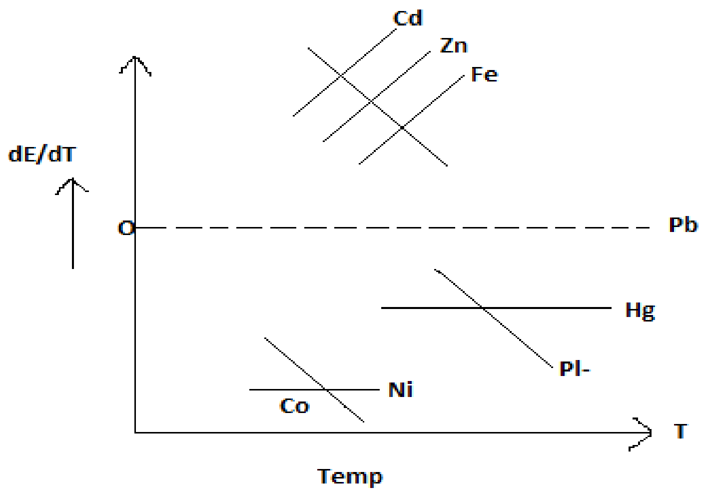

Fig.2.7

2.13: Uses of Thermo – electric power diagrams

(i) Determination of Total *emf*

MN represents the thermo – electric power line of a metal like copper coupled with lead. MN has a +ve slope (Fig.2.8).

Let A and B be two points corresponding to temperatures T_1 K and T_2 K respectively.

Consider a small strip abdc of thicknes dT with junctions maintained at temperatures T and (T+dT).

The *emf* developed when the two junctions of the thermocouple differ by dT is

$$dE = dT\left(\frac{dE}{dT}\right) = \text{area abcd}$$

Total *emf* developed when the junctions of the couple are at temperatures T_1 and T_2 is

$$E_s = \int_{T_1}^{T_2} dT\ \left(\frac{dE}{dT}\right) \quad = \text{area ABDC}$$

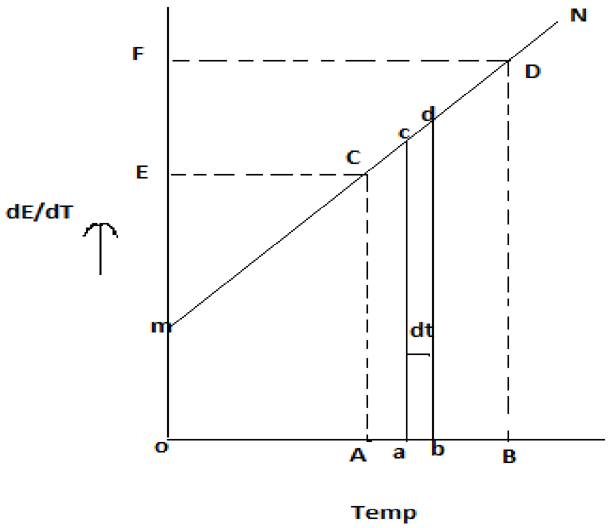

Fig.2.8

ii) Determination of Peltier emf

Let π_1 and π_2 be the Peltier coefficients for the junctions of the couple at temperature T_1 and T_2 respectively.

The Peltier coefficient at the hot junction (T_2) is

$$(\pi_2) = T_2\left(\frac{dE}{dT}\right)_{T_2} = \text{OB x BD} = \text{area OBDF.}$$

Similarly, Peltier coefficient at the cold junction (T_1) is

$$\pi_1 = T_1\left(\frac{dE}{dT}\right)_{T_1} = \text{OA x AC} = \text{area OACE.}$$

π_1 and π_2 give the Peltier *emf*s at T_1 and T_2. Peltier *emf* between temperatures T_1 and T_2 is

$$E_p = \pi_2 - \pi_1 = \text{area OBDF} - \text{area OACE}$$

$$= \text{area ABDFECA}$$

(iii) Determination of Thomson *emf*

Total *emf* developed in a thermocouple between temperature T_1 and T_2 is

$$E_s = (\pi_2 - \pi_1) + \int_{T_1}^{T_2} (\sigma_a - \sigma_b)\, dT$$

Here σ_a and σ_b represent the Thomson coefficients of two metals constituting thermocouple.

If the metal A is copper and B is Lead, then $\sigma_B = 0$.

$$\therefore \quad E_s = (\pi_2 - \pi_1) + \int_{T_1}^{T_2} (\sigma_A)\, dT$$

(or) $$\int_{T_1}^{T_2} (\sigma_A)\, dT = -\left[(\pi_2 - \pi_1) - E\right]$$

Thus, the magnitude of Thomson *emf* is given by

$$E_{Th} = (\pi_2 - \pi_1) - E \qquad \text{Area ABDFECA – Area ABDC}$$

= Area CDFE

iv) Thermo emf in a general couple neutral temperature and temperature of inversion

Let us consider a thermo couple consisting of any two metals, say Cu and Fe. AB and CD are the thermo – electric power lines for Cu and Fe with respect to lead (Fig.2.9)

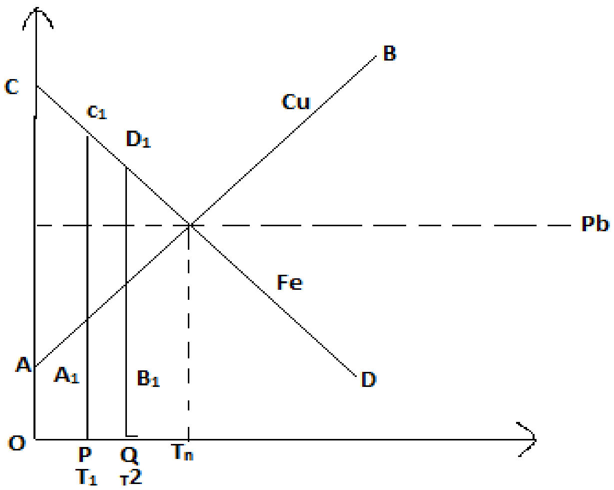

Fig.2.9

Let T_1 and T_2 be the temperature of the cold and hot junctions corresponding to P and Q.

Emf of Cu – Pb Thermocouple = area PQB_1A_1

Emf of Fe – Pb Thermocouple = area PQD_1C_1

Emf of CU- Fe thermocouple is

$$E_{Cu}^{Fe} = \text{Area } PQD_1C_1 - \text{Area } PQB_1A_1$$

$$= \text{Area } A_1B_1D_1C_1$$

T_n is called the neutral temperature at which two power lines intersect with each other.

Suppose temperatures of the junctions, T_1 and T_2 for a Cu-Fe thermocouple are such that the neutral temperature T_n lies between T_1 and T_2 (Fig.2.10). Then the thermo *emf* will be represented by the difference between the areas A_1NC_1 and B_1D_1N because these areas represent opposing emf's. In particular case when $T_n = (T_1 + T_2)/2$. These areas are equal and the resultant *emf* is zero. In this case T_2 is the temperatre of inversion" for Cu-Fe thermocouple.

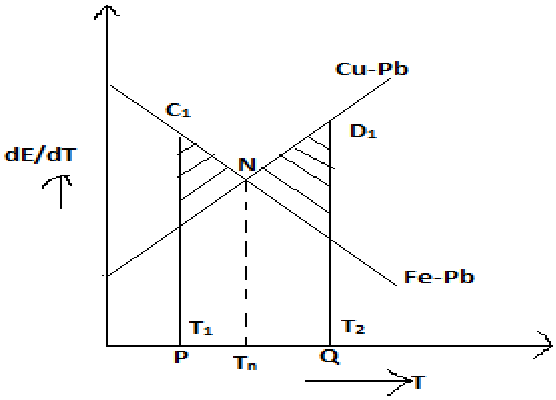

Fig.2.10

2.14: Applications of thermo electric effect :-

The important applications of thermoelectric effects are,

1. Thermopile :-

The thermopile consists of a number of small strips of antimony and bismuth placed alternately as shown in the Fig. (2.11)

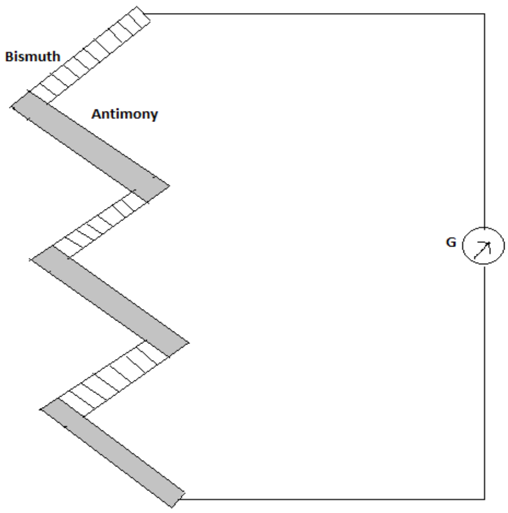

Fig.2.11

The ends are soldered and the combination is so arranged that the bismuth-antimony junctions lie at one side forming the hot junction and the antimony – bismuth junction lie at the opposite side forming the cold junction. A sensitive Galvanometer is included in series with the pile. When the hot junction is exposed to the Thermal radiation. The rise in temperature produces a thermo-electric current in the circuit, which in turn, produces a deflection in the galvanometer. In the actual construction, the strips are arranged in the form of a cube with all the hot junctions forming one face of the cube and all the cold junctions forming the opposite face. The different layers of the strips are insulated with paraffin paper or mica. The thermopile is mounted on a vertical stand and is provided with a conical protector to avoid stray radiations.

2). Boy's radio micrometer:

The Boy's radio micrometer is a highly sensitive instrument to measure the amount of thermal radiations.

It consists of Antimony – Bismuth (A and B in Fig: 2.12) thermocouple with the lower junction is contact with a blackened platinum or copper disc (D). The thermocouple circuit is completed by a single loop of copper wire. The copper is suspended between the poles N and S of a powerful horse – shoe magnet by means of fine quartz fibre and a glass rod. The suspension fibre is provided with a small mirror to measure the angular deflection of the loop by the lamp and magnetic field just as in the case of a suspended moving coil galvanometer. The angular deflection produced is directly proportional to the quantity of thermal radiation falling per sec on the platinum disc.

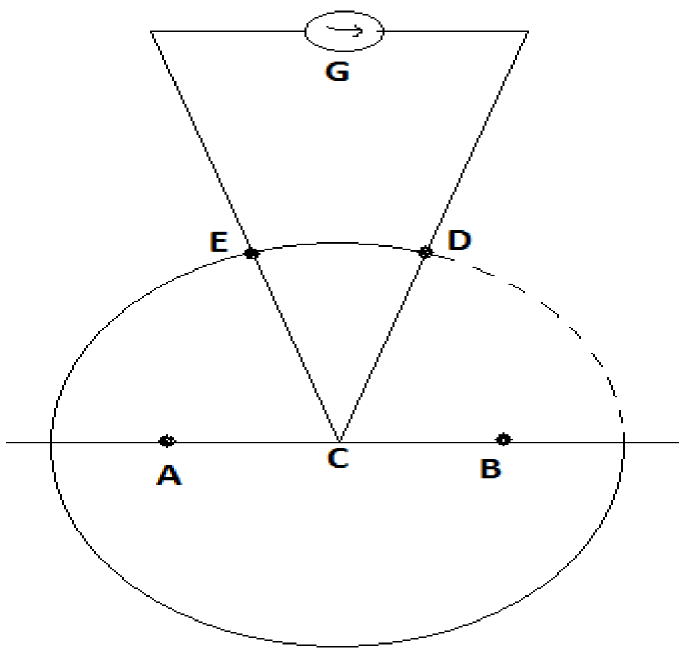

Fig.2.12

3. **Thermo – milli ammeter**

It is devised by Sir, J.A. Fleming. It is a very sensitive ammeter and it is used for measuring both alternating and direct continuous current. This instrument consists of a fixed wire of Constantan. One junction of a Bismuth – Tellurium thermocouple is soldered to a Constantan wire at C, where as the other junction is connected scale arrangement. A magnet screen (not shown in the Fig:2.13) surrounds the lower part of the instrument to protect the diamagnetic bismuth from the field of the horse – shoe magnet. The whole system is provided with a brass case to protect it from air currents.

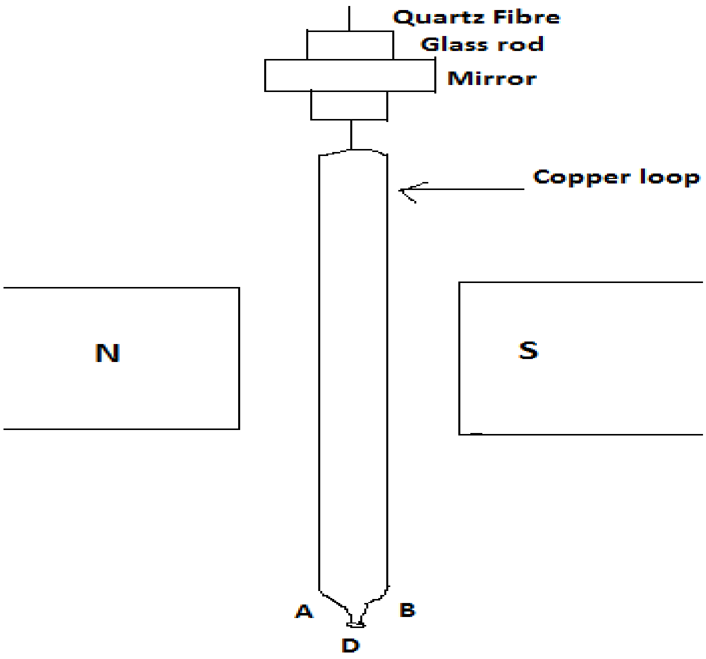

Fig.2.13

When the thermal radiation is incident on the blackened platinum disc., a thermo electric current is set up in the circuit and this, in turn, rotates the copper loop in the a galvanometer by means of leads E and D. The current to be measured is allows to pass through the wire AB. As a result, a deflection is obtained in the galvanometer due to the thermo electric current developed.

The unknown current corresponding to the observed deflection can be directly read from a pre obtained calibration curve. The calibration curve is obtained by sending known currents through AB and observing the resultant deflections. For better sensitiveness, the thermo electric part of the apparatus is enclosed in an evacuated glass bulb.

UNIT – III

Transient Currents

3.1: Transient Phenomena

Transient phenomena are phenomena that exist only for short a while and are not simple periodic functions of time.

3.2: Transient currents and voltages

Production of currents and voltages in a circuit in a very short interval of time is called transient currents and voltages. During the transient process, the currents and voltages in the circuit are functions of time.

3.3: Growth and decay of current in L-R circuit

(i) Growth of current in L-R circuit.

Consider a circuit consisting of a battery of a steady *emf* E, an inductance L and a resistance R as shown in Fig.3.1.

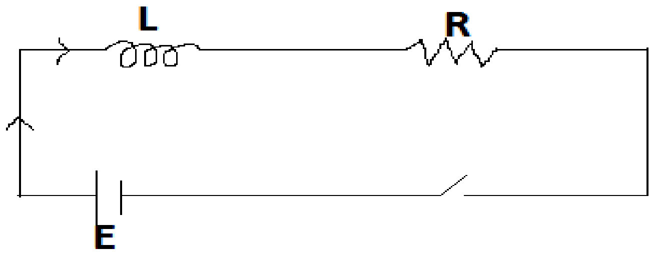

Fig.3.1

When the key is suddenly pressed, there is growth of current in the circuit and a back *emf* is induced. Let I be the current at any instant of time t then

$$E = RI + L\frac{dI}{dt} \qquad (3.1)$$

When the current reaches maximum value I_0, the back *emf* $L\frac{dI}{dt} = 0$.

$$\therefore E = RI_0 \qquad (3.2)$$

From eqns (3.1) and (3.2), we have,

$$RI_0 = RI + L\frac{dI}{dt}$$

$$R(I_0 - I) = L\frac{dI}{dt} \qquad (3.3)$$

Taking $(I_0 - I) = x$

Differentiating with respect to time, we get

$$\frac{-dI}{dt} = \frac{dx}{dt}$$

Substituting this in eq. (3.3), we get

$$Rx = -L\frac{dx}{dt}$$

$$\frac{dx}{x} = -\frac{R}{L}dt$$

Integrating,

$$\log_e x = -\frac{R}{L}t + k$$ where k is a constant

$$\therefore \log_e (I_0 - I) = -\frac{R}{L}t + k$$

when t = 0, I = 0,

$$\therefore \log_e I_0 = k$$

$$\therefore \log_e (I_0 - I) = -\frac{R}{L}t + \log_e I_0$$

$$\log_e (I_0 - I) - \log_e I_0 = -\frac{R}{L}t$$

$$\log_e \left(\frac{I_0 - I}{I_0}\right) = -\frac{R}{L}t$$

$$\frac{I_0 - I}{I_0} = e^{\frac{-R}{L}t}$$

(or) $$1 - \frac{I}{I_0} = e^{\frac{-R}{L}t}$$

or, $$\frac{I}{I_0} = 1 - e^{\frac{-R}{L}t}$$

$$I = I_0 (1 - e^{\frac{-R}{L}t}) \qquad (3.4)$$

The quantity L/R is called the time constant of the circuit.

Time constant

The quantity $\frac{L}{R}$ has the dimension of time and is called the time constant (λ) of the L-R circuit.

If $\frac{L}{R}$ = t, then I = I_0 (1-e^{-1}) = I_0 (1- $\frac{1}{e}$) = 0.632 I_0.

Thus,

The time constant L/R of a L-R circuit is the time taken by the current to grow from zero to 0.632 times the maximum value of current - I_0 in the circuit.

The graph between current and time at the time of the growth of current is shown in Fig.3.2.

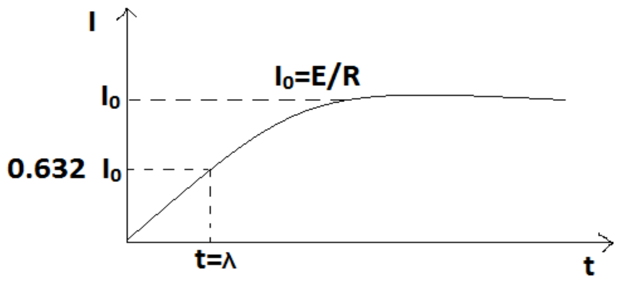

Fig.3.2

3.4: Decay of current in a circuit containing L and R

When the circuit is broken, an induced *emf*, equal to -L$\frac{dI}{dt}$ is again produced in the inductance L and it slows down and decay to zero. The current in the circuit decays from maximum value I_0 to zero. During the decay, let I be the current at time t. In this case E=0.

The *emf* equation for the decay of current is

$$0 = RI + L\frac{dI}{dt} \qquad (3.5)$$

$$\therefore \frac{dI}{I} = -\frac{R}{L}dt$$

Integrating, $\log_e I = -\frac{R}{L}t + k$, where k is a constant.

when t=0, I=I_0, $\therefore \log_e I_0 = k$

$$\therefore \log_e I = -\frac{R}{L}t + \log_e I_0$$

(or) $$\log \frac{I}{I_0} = \frac{-R}{L} t$$

or, $$\frac{I}{I_0} = e^{\frac{-R}{L} t}$$

$$\therefore\ I = I_0 \ e^{\frac{-R}{L} t} \qquad (3.6)$$

Eq.(3.6) represents the current at any instant t during decay. A graph between current and time during decay is shown in Fig. 3.3.

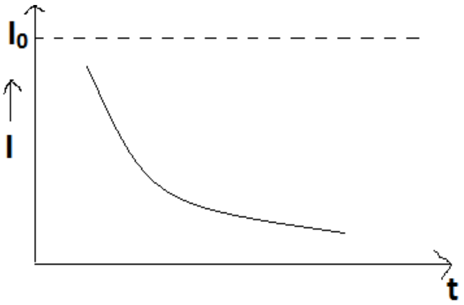

Fig.3.3

Time constant:

$$t = \frac{L}{R},\ \therefore I = I_0 e^{-1} = \frac{1}{e}\ I_0 = 0.365\ I_0$$

$\therefore$ The time constant $\frac{L}{R}$ of a L-R circuit may also be defined as the time in which the current in the circuit falls to $\frac{1}{e}$ of its maximum value when *emf* is removed.

Fig .3.4 shows that the growth and decay curves are complementary with each other.

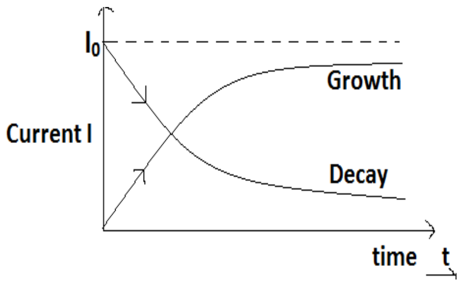

Fig.3.4

3.5: Charge and Discharge of a capacitor through a Resistor

(i) Growth of charge.

Consider a circuit consisting of a cell of *emf* E, a key K, a capacitor C and a resistance R as shown in Fig. 3.5.

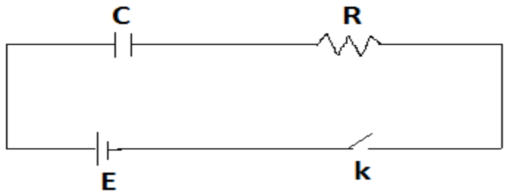

Fig.3.5

When the current is started, let q be the instantaneous charge on the condenser and I be the instantaneous current.

$\therefore$ The *emf* equation for the CR circuit is

$$RI + \frac{q}{c} = E$$

But $\quad I = \frac{dq}{dt} =$ rate of flow of charge,

Hence, $$R\frac{dq}{dt} + \frac{q}{c} = E$$

(or) $$R\frac{dq}{dt} = E - \frac{q}{c}$$

and $$E = \frac{q_0}{c}$$

(or) $$R\frac{dq}{dt} = \frac{q_0}{c} - \frac{q}{c}$$

$$R\frac{dq}{dt} = \frac{(q_0 - q)}{c}$$

$$\therefore \frac{dq}{(q_0 - q)} = \frac{dt}{CR}$$

Integrating, - $$-\log_e (q_0 - q) = \frac{t}{CR} + K$$

Where K is a constant

Applying the initial condition,

When t=o, q=o, $-\log_e q_0 = K$

$$\therefore -\log_e (q_0 - q) = \frac{t}{CR} - \log_e q_0$$

(or) $$\log_e (q_0 - q) - \log_e q_0 = -\frac{t}{CR}$$

$$\log_e \left(\frac{(q_0 - q)}{q_0} \right) = \frac{-t}{CR}$$

$$\frac{q_0 - q}{q_0} = e^{-t/CR}$$

(or) $$1 - \frac{q}{q_0} = e^{-t/CR}$$

(or) $$\frac{q}{q_0} = 1 - e^{-t/CR}$$

(or) $$\boxed{q = q_0 (1 - e^{-t/CR})} \qquad (3.7)$$

This is called instantaneous value of the charge at time t. The term CR is called time constant of the circuit.

Time constant

At the end of time t = CR,

$\therefore$ the Eq. (3.7) becomes, $\quad q = q_0 (1 - e^{-t/t})$

$$= q_0 (1 - e^{-1}) = q_0 (1 - \frac{1}{e})$$

$$\boxed{\therefore q = 0.632 \, q_0}$$

Thus, the time constant may be defined as the time taken by the capacitor to get charged to 0.632 times its maximum value.

The growth of charge is shown in Fig.3.6

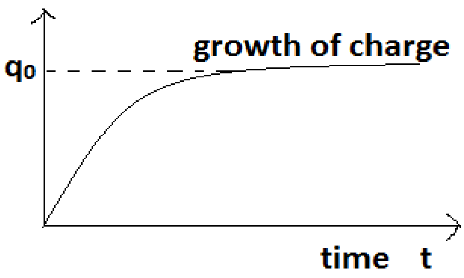

Fig.3.6

3.6: Decay of charge (Discharging of a capacitor though Resistance)

Let the capacitor having charge q_0 be now discharged by opened the key K. The charge flows out of the capacitor. In this case E = 0.

The *emf* equation is

$$R \frac{dq}{dt} + \frac{q}{c} = 0$$

(or) $$\frac{dq}{dt} = -\frac{1}{CR} dt$$

Integrating, $\log_e q = \frac{-t}{CR} + K,$ where K is a constant

Initial condition, when t = 0, q = q_0,

$$\therefore \log_e q_0 = K$$

$$\therefore \log_e q = \frac{-t}{CR} + \log_e q_0$$

Or, $$\therefore \log_e q - \log_e q_0 = \frac{-t}{CR}$$

$$\therefore \log_e \frac{q}{q_0} = \frac{-t}{CR}$$

Or $$\frac{q}{q_0} = e^{-t/CR}$$

$$\boxed{q = q_0 \ e^{-t/CR}} \qquad (3.8)$$

This is called the instantaneous value of the charge during the discharge. The graph for decay of charge is shown in Fig.3.7.

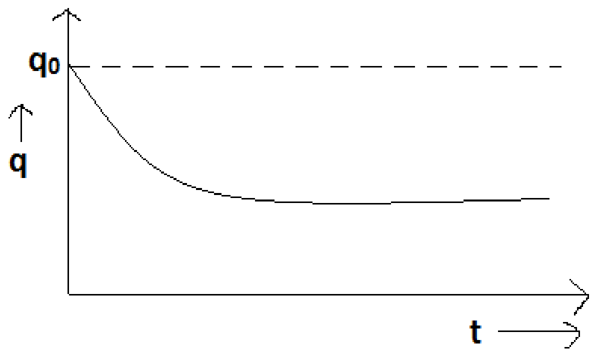

Fig.3.7

Time constant: If we put t = CR in Eq.(3.8), we get q = q_0e^{-1}

Ie q = q_0 x 1/e, q = 0.368 q_0

Hence time constant may also be defined as the time taken by the capacitor to discharge the charge from 0.368 of its maximum value.

3.7 Grow of charge in a circuit with inductance, capacitance and resistance

Consider a circuit containing an inductance L, capacitance C and resistance R joined in series to a cell of *emf* E (Fig.3.8). When the key is pressed, the capacitor is charged. Let Q be the charge on the capacitor and I the current in the circuit at an instant t during charging. Then, the P.d. across the capacitor is $\frac{Q}{C}$ and the self induced *emf* in the inductance coil is $L\left(dI/dt\right)$, both being opposite to the direction of E. The P.d a cross the resistance R is RI.

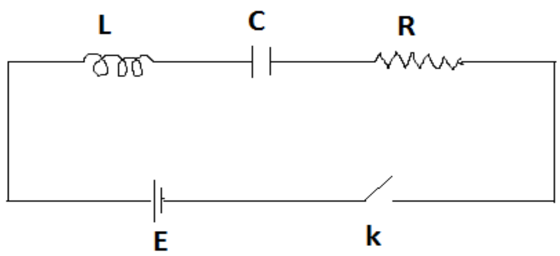

Fig.3.8

The equation of *emf* is

$$L\frac{dI}{dt}+RI+\frac{Q}{C}=E \qquad (3.9)$$

But

$$I=\frac{dQ}{dt} \text{ and } \frac{\text{dI}}{\text{dt}}=\frac{d^2Q}{dt^2}$$

$$\therefore L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=E$$

Or
$$\frac{d^2Q}{dt^2}+\frac{R}{L}\frac{dQ}{dt}+\frac{Q-CE}{LC}=0$$

Putting $\frac{R}{L}=2b$ and $\frac{1}{LC}=K^2$ we have

$$\frac{d^2Q}{dt^2}+2b\frac{dQ}{dt}+K^2(Q-CE)=0 \qquad (3.10)$$

Let $x = Q\text{-}CE$, Then $\frac{dx}{dt}=\frac{dQ}{dt}$ *and* $\frac{d^2x}{dt^2}=\frac{d^2Q}{dt^2}$

Eq. (3.10) becomes,
$$\frac{d^2x}{dt^2}+2b\frac{dx}{dt}+K^2x=0 \qquad (3.11)$$

Hence the most general solution of Eq. (3.11) is

$$x=Ae^{\left[-b+\sqrt{b^2-k^2}\right]t}+Be^{\left[-b-\sqrt{b^2-k^2}\right]t}$$

Now CE = Q_0 = final steady charge on the capacitor.

$$\therefore x = Q-CE = Q-Q_0$$

Hence
$$Q-Q_0=Ae^{\left[-b+\sqrt{b^2-k^2}\right]t}+Be^{\left[-b-\sqrt{b^2-k^2}\right]t}$$

(or) $$Q = Q_0 + Ae^{\left[-b+\sqrt{b^2-k^2}\right]t} + Be^{\left[-b-\sqrt{b^2-k^2}\right]t} \quad (3.12)$$

Using initial conditions

at t= 0, Q = 0

$\therefore$ 0 = Q_0 + (A+B)

Or A + B = - Q_0 (3.13)

$$\frac{dQ}{dt} = A\left[-b+\sqrt{b^2-k^2}\right] e^{\left[-b+\sqrt{b^2-k^2}\right]t} + B\left[-b-\sqrt{b^2-k^2}\right] e^{\left[-b-\sqrt{b^2-k^2}\right]t}$$

At t = 0, $\frac{dQ}{dt} = 0$

$$0 = A\left[-b+\sqrt{b^2-k^2}\right] + B\left[-b-\sqrt{b^2-k^2}\right]$$

$$\sqrt{b^2-k^2}\,[A-B] = b[A+B] = -bQ_0$$

(or) $$A - B = -\frac{Q_0 b}{\sqrt{b^2-k^2}} \quad (3.14)$$

Solving eqs (3.13) and (3.14)

$$A = -\frac{1}{2}Q_0\left(1+\frac{b}{\sqrt{b^2-k^2}}\right) \quad (3.15)$$

$$B = -\frac{1}{2}Q_0\left(1-\frac{b}{\sqrt{b^2-k^2}}\right) \quad (3.16)$$

Substituting the values of A and B in Eq. (3.12), we have

$$Q = Q_0 - \frac{1}{2}Q_0\, e^{-bt}\left[\left(1+\frac{b}{\sqrt{b^2-k^2}}\right) e^{\sqrt{(b^2-k^2)}t} + \left(1-\frac{b}{\sqrt{b^2-k^2}}\right) e^{-\sqrt{(b^2-k^2)}t}\right]$$

(3.17)

Case 1:-

If $b^2 > k^2$, $\sqrt{b^2-k^2}$ is real. The charge on the capacitor grows experientially with time and attains the maximum value Q_0 asymptotically (curve 1 of Fig.3.9). The charge is known as *over damped* or *dead beat.*

Case ii :-

If $b^2=k^2$, the charge rises to a maximum value Q_0 in a short time (curve 2 in Fig.3.9). Such a charge is called *critically damped***.**

Case iii:-

$b^2 < k^2$, $\sqrt{b^2-k^2}$ is imaginary.

Let $\sqrt{b^2-k^2} = i\omega$, where $i=\sqrt{-1}$ and $\omega=\sqrt{k^2-b^2}$

Eq. (3.17) may be written as

$$Q=Q_0-\frac{1}{2}Q_0\, e^{-bt}\left[\left(1+\frac{b}{i\omega}\right)e^{i\omega t}+\left(1-\frac{b}{i\omega}\right)e^{-i\omega t}\right]$$

$$Q=Q_0-Q_0\, e^{-bt}\left[\left(\frac{e^{i\omega t}+e^{i\omega t}}{2}\right)+\frac{b}{\omega}\left(\frac{e^{i\omega t}+e^{-i\omega t}}{2i}\right)\right]$$

$$Q=Q_0-Q_0\, e^{-bt}\left(\cos\omega t+\frac{b}{\omega}\sin\omega t\right)$$

$$Q=Q_0\left[1-\frac{e^{-bt}}{\omega}\left(\omega\cos\omega t+b\,\sin\omega t\right)\right]$$

Let $\omega = k\sin\alpha$ and $b = k\cos\alpha$ so that $\tan\alpha = \omega/b$.

$$Q=Q_0\left[1-\frac{e^{-bt}}{\omega}\left(k\sin\alpha\cos\omega t+k\cos\alpha\sin\omega t\right)\right]$$

or

$$Q=Q_0\left[1-\frac{k\,e^{-bt}}{\omega}\left(\sin\left(\omega t+\alpha\right)\right)\right] \tag{3.18}$$

$$Q=Q_0\left[1-\frac{e^{-\frac{R}{2L}t}\sqrt{\frac{1}{LC}}}{\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}}\sin\left[\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\right)t+\alpha\right]\right]$$

This equation represents a damped oscillatory charge as shown by the curve (Fig.3.9). The frequency of the oscillation in the circuit is given by

$$\gamma=\frac{\omega}{2\pi}=\frac{\sqrt{k^2-b^2}}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$$

When R = 0,

$$\gamma = \frac{1}{2\pi\sqrt{LC}}$$

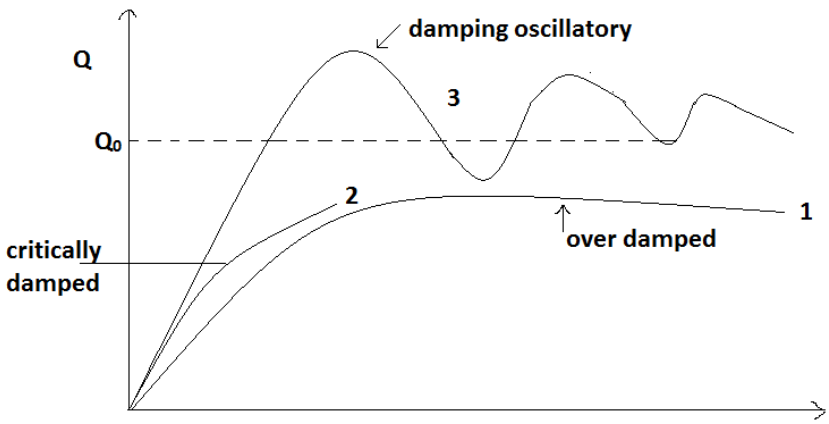

Fig.3.9

3.8: Discharge of a Capacitor through an Inductor and a Resistor in series (Decay of charge in LCR circuit)

Consider a circuit containing a capacitor of capacitance C, an inductance L and a resistance R joined in series (Fig.3.10). E is a cell. k_2 is kept open. The capacitor charged to maximum charge Q_0 by closing the key k_1. On opening k_1 and closing key k_2, the capacitor discharges through the inductance L and resistance R.

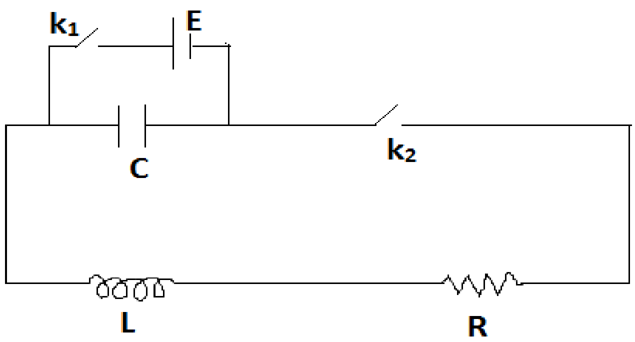

Fig.3.10

Let I be the current in the circuit and Q be the charge in the capacitor at any instant during discharge. The circuit equation then is

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = 0$$

But $$I = \frac{dQ}{dt} \text{ and } \frac{dI}{dt} = \frac{d^2Q}{dt^2}$$

$$\therefore L \frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0 \qquad (3.19)$$

Let $$\frac{R}{L} = 2b \text{ and } \frac{1}{LC} = K^2 \quad \text{than}$$

$$\frac{d^2Q}{dt^2} + 2b\frac{dQ}{dt} + K^2 Q = 0 \qquad (3.20)$$

The General solution of this equation is

$$Q = Ae^{(-b+\sqrt{b^2-k^2})t} + Be^{(-b-\sqrt{b^2-k^2})t} \qquad (3.21)$$

Where A and B are arbitrary constants

When t = o, Q = Q_0 and from Eq. (3.21), A+ B = Q_0 (3.22)

$$\frac{dQ}{dt} = A\left[-b+\sqrt{b^2-k^2}\right] e^{\left[-b+\sqrt{b^2-k^2}\right]t} + B\left[-b-\sqrt{b^2-k^2}\right] e^{\left[-b-\sqrt{b^2-k^2}\right]t}$$

When $$t = 0, \quad \frac{dQ}{dt} = 0$$

$$\therefore \; A(-b+\sqrt{b^2-k^2}) + B(-b-\sqrt{b^2-k^2}) = 0$$

$$-b(A+B) + \sqrt{b^2-k^2}\,(A-B) = 0$$

$$-bQ_0 + \sqrt{b^2-k^2}\,(A-B) = 0$$

$$\therefore \; A - B \; -\frac{bQ_0}{\sqrt{b^2-k^2}} \qquad (3.23)$$

From Eqs. (3.22) and (3.23) we get,

$$A = \frac{1}{2}Q_0\left(1+\frac{b}{\sqrt{b^2-k^2}}\right) \text{ and } B = \frac{1}{2}Q_0\left(1-\frac{b}{\sqrt{b^2-k^2}}\right)$$

Putting these values of A and B in Eq. (3.21), we get,

$$Q = \frac{1}{2} Q_0 \, e^{-bt} \left[\left(1 + \frac{b}{\sqrt{b^2 - k^2}} \right) e^{\sqrt{(b^2 - k^2)t}} + \left(1 - \frac{b}{\sqrt{b^2 - k^2}} \right) e^{-\sqrt{(b^2 - k^2)t}} \right] \quad (3.24)$$

Case- i :-

If $b^2 > k^2$, $\sqrt{b^2 - k^2}$ is real and positive and the charge of the capacitor decays exponentially, becoming zero asymptotically. This discharge is known as *over damped, non oscillatory or dead beat.*

Case (ii) :- when $b^2 = k^2$, $Q = Q_0 \, e^{-bt}$.

This represents a non – oscillatory discharge. This discharge is known as *critically damped* . The charge decreases to zero experientially in a short time.

Case iii :-

If $b^2 < k^2$, $\sqrt{b^2 - k^2}$ is imaginary.

$$\sqrt{b^2 - k^2} = i\omega \text{ where } \omega = \sqrt{b^2 - k^2}$$

$$\therefore Q = \frac{1}{2} Q_0 \, e^{-bt} \left[\left(1 + \frac{b}{i\omega} \right) e^{i\omega t} + \left(1 - \frac{b}{i\omega} \right) e^{-i\omega t} \right]$$

$$= Q_0 \, e^{-bt} \left[\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2} \right) + \frac{b}{\omega} \left(\frac{e^{i\omega t} - e^{i\omega t}}{2i} \right) \right]$$

$$= \frac{Q_0 \, e^{-bt}}{\omega} \left(\omega \cos \omega t + \frac{b}{3} \sin \omega t \right)$$

Let $\omega = \sin \alpha$ and $b = k \cos \alpha$, so that $\tan \alpha = \omega / b$.

$$Q = \frac{Q_0 \, e^{-bt} k}{\omega} (\cos \omega t \, \sin \alpha + Cos \, \alpha \sin \omega t)$$

$$= \frac{Q_0 \, e^{-bt} k}{\omega} \sin(\omega t + \alpha)$$

$$Q = \left[\frac{Q_0 e^{-\frac{-R}{2L} t}}{\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2} \right)} \sqrt{LC}} \sin \left[\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2} \right)} t + a \right] \right]$$

This equation represents a damped oscillatory charge. The charge oscillates above and below zero till it finally settles down to zero value.

The frequency of oscillation in the circuit in given by

$$\gamma = \frac{\omega}{2\pi} = \frac{\sqrt{k^2 - b^2}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

When R = 0,

$$\gamma = \frac{1}{2\pi\sqrt{LC}}$$

The condition for oscillatory discharge is

$$\frac{R^2}{4L^2} < \frac{1}{\mathrm{LC}} \, (or) \, R < 2\sqrt{L/C}$$

3.9 Measurement of High resistance by Leakage

When a capacitor of capacitance C and initial charge Q_0, is allowed to discharge through a resistance R for a time t, the charge remainting on the capacitor is given by

$$Q = Q_0 \, e^{-t/CR}$$

$$Q/Q_0 = e^{-t/CR}$$

$$\therefore \log_e\left(\frac{Q_0}{Q}\right) = \frac{t}{CR}$$

$$\therefore R = \frac{t}{C\log_e\left(\frac{Q_0}{Q}\right)} = \frac{t}{2.3026\,C\log_{10}\left(\frac{Q_0}{Q}\right)}$$

If R is high, CR will be high and the rate of discharge of capacitor will be very slow. Thus if we determine Q_0/Q from experiment, then R can be calculated.

Connections are made as shown in Fig.3.11. C is a capacitor of known capacitance. R is the high resistance to be measured. B.G is a ballishc galvanometer. E is a cell and K_1, K_2, K_3 tap keys. Keeping K_2 and K_3 open, the capacitor is charged by depressing the key K_1. K_1 is then opened and at once K_3 is closed. The capacitor discharges through the galvanometer, which records a throw θ_0. The throw θ_0 is proportional to Q_0. The capacitor is again charged to the maximum value keeping K_2 and K_3 open and closing K_1.

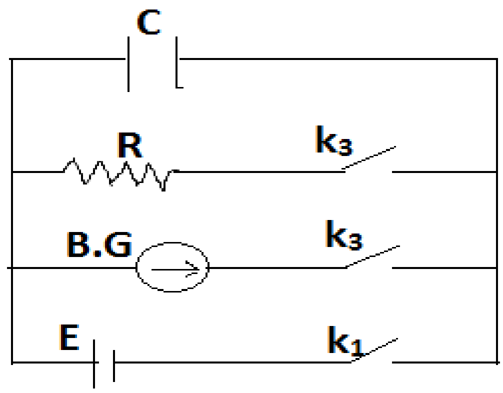

Fig.3.11

K_1 is then opended and K_2 is closed for a known time t. Some of the charge leaks through R. K_2 is oponed and at once K_3 is closed. The charge Q remaining on the capacitor then discharges through the galvanometer. The resulting throw θ is noted.

The $Q \propto \theta$ Now $\frac{Q_0}{Q} = \frac{\theta_0}{\theta}$

$$\therefore R = \frac{t}{2.3026 \log_{10} \frac{\theta_0}{\theta}}$$

A series of values of t and θ are obtained. A graph is plotted between t and $\log_{10}\left(\frac{\theta_0}{\theta}\right)$ which is a straight line. Its slope gives the mean value of t/ $\log_{10}\left(\frac{\theta_0}{\theta}\right)$

As C is known the value of R can be calculated.

UNIT – IV

j - Operator Method

4.1: Use of Operator j in study of A.C. circuits

The operator **j** is defined as a quantity which is numerically equal to $\sqrt{-1}$, and which represents the rotation of a vector through $90°$ in anti – clockwise direction – **j** represents the rotation of a vector through $90°$ in clockwise direction.

We know that in A.C circuits, E_L and E_C always lie at $90°$ in anti clockwise and clockwise direction respectively with respect to E_R (Fig.4.1).

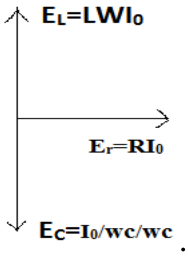

Fig.4.1

Hence total *emf* of a circuit having L,C, R, will be

$$E = E_R + jE_L - jE_C$$

A source of alternating *emf* E is denoted by

$$E = E_0 \sin \omega t$$

Interms of complex form,

$$E = E_0 e^{j\omega t}$$

The instantaneous current in the A.C. circuit,

$$I = I_0 \sin(\omega t - \phi)$$

In complex form,

$$I = I_0 e^{j(\omega t - \phi)}$$

Since the voltage across the inductor leads the current passing through it by $90°$, the inductive reactance ωL can be written as $j\omega L$, *ie*.,,

$$Z_L = j\omega L = jX_L$$

Since the voltage across the capacitor lags the current passing through it by 90° the capacitive reactance $1/\omega C$ can be written as $-j/\omega C = 1/j\omega C$

$$i.e.,\ Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = -jX_C$$

A complex impedance can be written as the sum of a real term and imaginary term which are to be resistance and complex reactance respectively,

$z = R + jX$, where $X = X_L - X_C$ is the effective reactance of the circuit.

4.2: LCR Circuit – Series Resonance Circuit.

Consider a circuit containing an inductance L, a capacitance C and a resistance R joined in series (Fig.4.2).

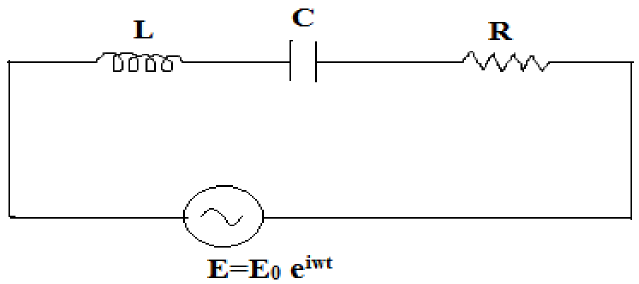

Fig.4.2

The series circuit is connected to an AC supply given by

$$E = E_0\, e^{j\omega t} \tag{4.1}$$

The total complex impedance is

$$Z = Z_R + Z_L + Z_C$$

$$= R + j(\omega L - \frac{1}{\omega C})$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\ e^{j\phi} \tag{4.2}$$

Where $$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Using ohm's law in complex form, the complex current in the circuit is

$$I = \frac{E}{Z} = \frac{E_0 e^{j\omega t}}{R^2 + (\omega L - 1/\omega C)^2 e^{j\phi}}$$

$$I = \frac{E_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \tag{4.3}$$

But
$$I_0 = \frac{E_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\therefore\ I = I_0\, e^{j(\omega t - \phi)} \tag{4.4}$$

The actual *emf* is the imaginary part of the equivalent complex *emf* . Hence the actual current in the circuit is obtained by taking the imaginary part of the above complex current.

$$\therefore i = \mathrm{Im}(I) = \frac{E_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \sin(\omega t - \phi) \tag{4.5}$$

The equivalent impedance of the series LCR circuit

$$\sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

The current 'lags' behind the voltage by an angle

$$\varphi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

4.3: Parallel Resonant Circuit :-

In this circuit, capacitor C is connected in parallel to the series combination of resistance R and inductance L. The combination is connected across the AC source. (Fig.4.3)

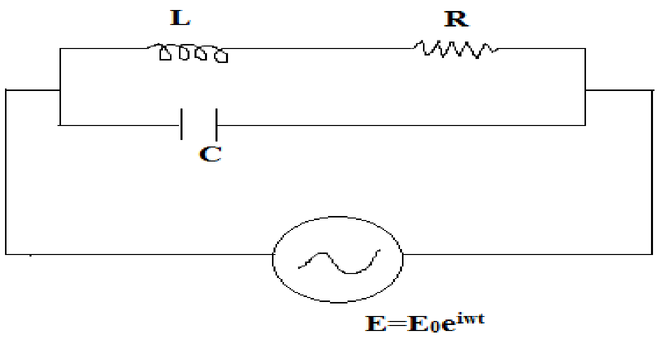

Fig.4.3

The applied voltage is

$$E = E_0\, e^{j\omega t}$$

The complex impedance of L-branch

$$Z_1 = R + j\omega L$$

Complex impedance of C – branch

$$Z_2 = \frac{1}{j\omega C}$$

Z_1 *and* Z_2 are parallel

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{R + j\omega L} + \frac{1}{1/j\omega C} = \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{(R - j\omega L)}{(R + j\omega L)(R - j\omega L)} + j\omega C$$

$$= \frac{R}{R^2 + (\omega L)^2} + j\left[\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right]$$

The current $I = E/Z = E \text{ x } \frac{1}{Z}$

$$\therefore I = E\left[\frac{R}{R^2 + (\omega L)^2} + j\left(\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right)\right]$$

Let $A\cos\phi = \frac{R}{R^2 + (\omega L)^2}$; $A\sin\phi = \omega C - \frac{\omega L}{R^2 + (\omega L)^2}$

$$\therefore I = E\ (A\cos\phi + j\,A son\phi) = E\,Ae^{j\phi} = E_0\ Ae^{j(\omega t + \phi)}$$

Where $\phi = \tan^{-1} - \frac{\omega C - \frac{\omega L}{(R^2 + (\omega L)^2)}}{\left(R / R^2 + (\omega L)^2\right)}$

$$A^2 = \frac{R^2}{\left(R^2 + \omega^2 L^2\right)^2} + (\omega C - \frac{\omega L}{R^2 + \omega^2 L^2})$$

The Magnitude of the admittance,

$$Y = \frac{1}{Z} = \frac{\sqrt{\left[R^2 + (\omega CR^2 + \omega L^2 C - (\omega L)^2\right]}}{R^2 + \omega^2 L^2}$$

The admittance will be minimum when

$$\omega CR^2 + \omega^3 L^2 C - \omega L = 0$$

or $\omega = \omega_0 = \sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$

or $$\gamma_0 = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$$

This is the resonant frequency of the circuit. If R is very small so that $\frac{R^2}{L^2}$ is negligible compared to $1/LC$

$$\gamma_0 = \frac{1}{2\pi\sqrt{(LC)}}$$

At such a minimum admittance, ie., maximum impedance, the circuit current is minimum. The graph between current and frequency is shown in Fig 4.4.

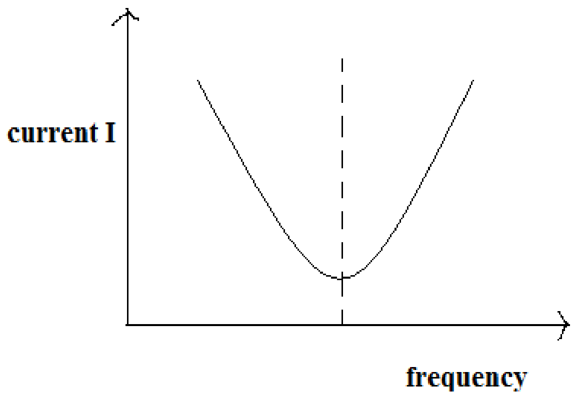

Fig.4.4

Impedance at Resonance :-

At resonance, $$Z = \frac{R^2 + (\omega L)^2}{R}$$

But $R^2 + (\omega L)^2 = \frac{L}{C}$ at resonance

$$\therefore Z = L/RC$$

Thus smaller the resistance R, larger is the impedance. If R is negligible, the impedance is infinite at resonance.

Rejecter circuit :-

The parallel resonant circuit does not allow the current of the same frequency as the natural frequency of the circuit. Thus it can be used to suppress the current of this particular frequency out of currents of many other frequencies. Hence the circuit is known as *rejector or filter circuit.*

Comparison between series and Parallel resonant circuit

Series resonance circuit		Parallel resonance circuit	
1.	An acceptor circuit	1.	A rejector circuit
2.	Resonant frequency $\gamma_r = \frac{1}{2\pi\sqrt{LC}}$	2.	Resonant frequency $\gamma_r = \frac{1}{2\pi\sqrt{LC}}$
3.	At resonance the impedance is a minimum equal to the resistance in the circuit.	3.	At resonance the impedance is maximum nearly equal to infinity.
4.	Selective	4.	Selective
5.	Used in the turning circuit to separate the wanted frequency from the incoming frequencies by offering low impedance at that frequency.	5.	Used to present a maximum impedance to the wanted frequency, usually in the plate circuit of value.

4.4: Power in AC circuit containing resistance, inductance and capacitance.

Consider an AC circuit containing resistance, inductance and capacitance, E and I vary continuously with time. Therefore power is calculated at any instant and then its mean is calculated over a completed cycle.

The instantaneous values of the voltage and current are given by

$$E = E_0 \sin \omega t$$

$$I = I_0 \sin (\omega t - \phi)$$

Where ϕ is the phase difference between current and voltage.

Hence power at any instant is

$$E \, x \, I = E_0 \, I_0 \sin \omega t \sin (\omega t - \phi)$$

$$= \frac{1}{2} E_0 \, I_0 \left[\cos \phi - \cos \, (2\omega t - \phi)\right]$$

Average power consumed over one complete cycle is

$$P = \frac{\int_0^T EI \, dt}{\int_0^T dt}$$

$$= \frac{\int_0^T \frac{1}{2} E_0 I_0 \left[\cos\phi - \cos(2\omega t - \phi)\right] dt}{T}$$

$$= \frac{1}{2}\frac{E_0 I_0}{T}\left[(\cos\phi)t - \frac{\sin(2\omega t - \phi)}{2\omega}\right]_0^T$$

$$= \frac{1}{2}\frac{E_0 I_0}{T}\left[(\cos\phi)T - 0 - \frac{\sin(2\omega t - \phi)}{2\omega} + \frac{\sin(-\phi)}{2\omega}\right]$$

Now $T = \frac{2\pi}{\omega}$ and $\sin(4\pi - \phi) = \sin(-\phi)$

$$P = \frac{1}{2}\frac{E_0 I_0 \omega}{2\pi}\left[(\cos\phi)\frac{2\pi}{\omega} - \frac{\sin(-\phi)}{2\omega} + \frac{\sin(-\phi)}{2\omega}\right]$$

$$= \frac{1}{2} E_0 I_0 \cos\phi$$

$$= \frac{E_0}{\sqrt{2}} \text{ x } \frac{I_0}{\sqrt{2}} \text{ x } \cos\phi$$

$$= E_{rms}\ I_{rms} \cos\phi$$

$$\text{average power} = (\text{Virtual volts}) \text{ x } (\text{Virtual ampere}) \text{ x } \cos\varphi$$

The term (virtual volts x virtual ampere) is called *apparent power* and $\cos\phi$ is called the *Power factor*.

Thus, $\text{True power} = \text{apparent power x Power factor}$

(or) the power factor is the ratio of the true power to the apparent power.

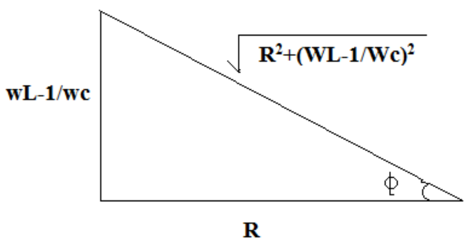

Fig.4.5

For a circuit containing resistance, capacitance and inductance in series,

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

From Fig.4.5, the expression for the power factor is

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Special cases :-

1. In a purely resistive circuit, $\varphi = 0,\ or\ \cos\phi = 1$

   $\therefore \text{true power} = E_v \text{ x } I_v$

2. In a purely inductive circuit, current lags behind the applied *emf* by 90^0 so that $\phi = 90°,\ \cos\phi = 0$

   Thus true power consumed = 0.

3. In a purely capacitive circuit, current leads the applied voltage by $90°$ so that $\phi = 90°$, or $\cos(-90) = \cos 90° = 0.$

   $\therefore \text{true power} = 0$

4. In an ac circuit containing a resistance and inductance in series,

   Power factor $\cos\phi = \dfrac{R}{\sqrt{R^2 + (\omega L)^2}}$

5. In an ac circuit contain a capacitance C and a resistance R in series,

$$\cos\phi = \frac{R}{\sqrt{\left(\frac{1}{\omega^2 C^2} + R^2\right)}}$$

4.5: Wattless Current

The average power dissipated during a complete cycle is $E_v I_v \cos\phi$.

"The current in A.C. circuit is said to be *Wattless*, when the average power consumed in the circuit is zero:.

If an ac circuit is purely inductive or purely capacitive with no ohmic resistance, angle $\phi = \pi/2$. So that $\cos\phi = 0$ or the power consumed is zero. The current in such a circuit does not perform any useful work and is rightly called the *Wattless* (or) *idle current.*

4.6: Choke coil

A Choke coil is an inductance coil which is used to control the current in an ac circuit.

Construction

A choke consists of a coil of several turns of insulated thick copper wire of low resistance but large inductance, wound over a laminated core (Fig.4.6). The core is layered and is made up of thin sheets of stalloy to reduce hysteresis losses. The laminations are coated with shellac to insulate and bound together firmly so as to minimise loss of energy due to eddy currents.

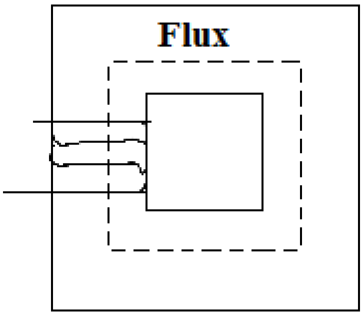

Fig.4.6

Principle :-

The average power dissipated in the choke coil is given by

$$P = \frac{1}{2} E_0 I_0 \cos\phi$$

The power factor $\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$

The inductance L of the choke coil is quite large on account of its large number of turns and the high permeability of iron core, while its resistance R is very small. Hence $\cos\phi$ is nearly zero. Therefore the power absorbed by the coil is extremely small. Thus the choke coil reduces the strength of the current without appreciable wastage of energy. The only waste of energy is due to the hysteresis loss in the iron core. The loss due to eddy currents is minimised by making the core laminated.

Uses :-

Chocking coils are very much used in electronic circuits, mercury lamps and sodium vapour lamps.

Preference of choke coil over an ohmic resistance – why?

The current in an ac circuit can also be diminished by using an ordinary ohmic resistance (rheostat) in the circuit. But such a method of controlling a.c. is not economical as much of the electrical energy $(I^2 Rt)$ supplied by the source is wasted as heat. Hence the choke coil is to be preferred over the ohmic resitance.

The energy used in establishing the magnetic field in the choke coil is restored when the magnetic field collapses. Hence to regulate a.c it is more economical to use a choke than a resistance.

4.7: Three phase A.C. Generators and Motors

A three phase alternator is shown in Fig. 4.7(a). It consists of three similar rectangular coils displaced equally from each ones, i.e. $120°$. Each oil is provided with its own brushes and slip rings.

Three *emf*s are generated when they are rotated at a constant velocity in a uniform magnetic field. They are of the same frequency and of equal values. Each of the three sources of voltage is called a 'phase'. Each phase voltage lags $120°$ behind that of the one preceding it (Fig. 4.7 (b), (c)).

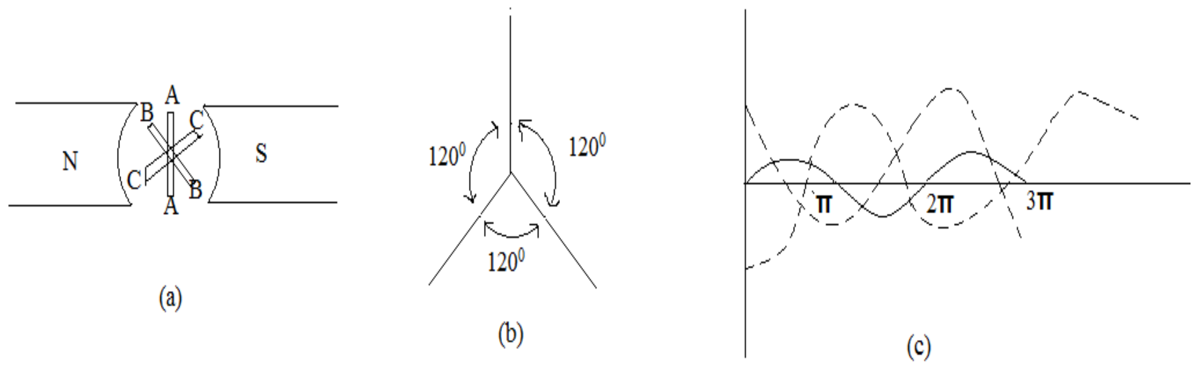

Fig.4.7

The instantaneous values of *emf* in each coil may be written as

$$E_1 = E_0 \sin \omega t,\ E_2 = E_0 \sin (\omega t + \frac{2\pi}{3}), E_3 = E_0 \sin (\omega t + \frac{4\pi}{3})$$

It can be used to supply a three phase system of three single phase circuits.

Advantages of 3-phase system:-

1. In 3-phase alternators the total power does not fluctuate, while in a single phase generator the current fluctuates.
2. The output power of a 3-phase alternator is always greater than that of a single phase generator of the same size.
3. Three phase system is superior for transmission and distribution of electrical energy. It involves lot of saving.

Frequency of A.C

The frequency of alternating *emf*, $v = nm$ where 'n' is the cycles of *emf* is generated per rotation and *m* is the rotations per see.

4.8 Distribution of three phase alternating current.

If three separate coils with angular separation of $120°$ are connected in the armature of an a.c dynamo, the voltages in the coils will have a phase difference of $120°$. This is called *3-phase a.c*. The three coils can be connected to three loads separately using six separate wires as shown in Fig.4.8. Then six heavy wires would be required in three separate single – phase systems. However, there are two methods of making connections by using only 4 and 3 wires.

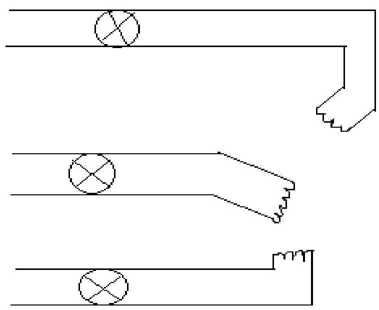

Fig.4.8

1. (i) Star Connection :

This method of transmission is used when all the phases are equally loaded. In case of balanced load, the neutral wire will be carrying three currents exactly similar but $120°$ out of phase with each other in a symmetrical 3-phase system. Their sum is zero. Hence neutral wire can be omitted and only three wires are required for transmission of 3-phase. Load can be put between any pair of phase QR, QS or SR (Fig.4.9)

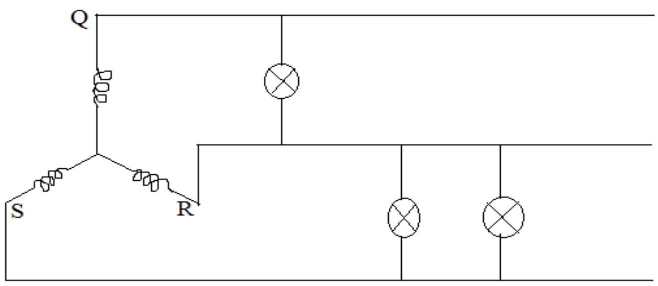

Fig.4.9

The *emf* between any line and the neutral given the *phase voltage* E_{ph}. The *emf* between two outer terminates is called *line voltage* V_L. In the star connection, the line voltage is $\sqrt{3}$ times the phase voltage. The phase difference between them is $30°$. The strength of line current is equal to the strength of phase current $I_L = I_{ph}$. The power is consumed in the circuit is three times the power per phase.

(ii) Three phase four wire system

When the load is unbalanced, ie., the different phases are unequally loaded, then 4-wire system is used instead of 3-wire system. The neutral or star points is connected to a wire called *neutral line***.** Three lines are taken from the free ands of Q, R and S and are called the *phase lines* (Fig.4.10 (a). The P.d between phase line is = $\sqrt{3}$ times the voltage between the phase and neutral point. For household supply, only one phase line and a neutral wire are connected (Fig 4.10(b)). For power supply in factories three phase wires and a neutral wire are connected to the factory.

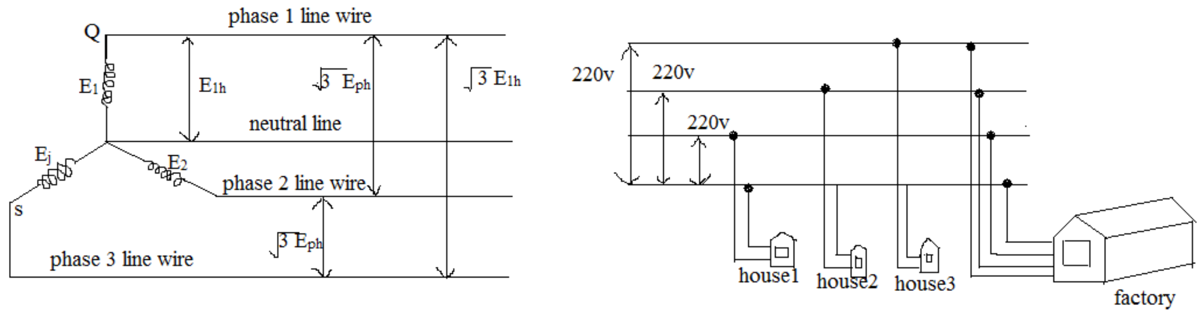

Fig.4.10

2. Delta connection

The delta connection is shown in Fig.4.11. Here the end of each winding is connected to the beginning of the next one, so that they form a closed triangle. In this type of connection, the line voltage of a generator is equal to its phase voltage. The line current is $\sqrt{3}$ times the phase current.

Output can be taken from QR, RS or QS. The windings may be delta connected only when the load on the phases is the same or almost the same. Otherwise the machinery may be damaged by strong currents in the closed circuit of the windings.

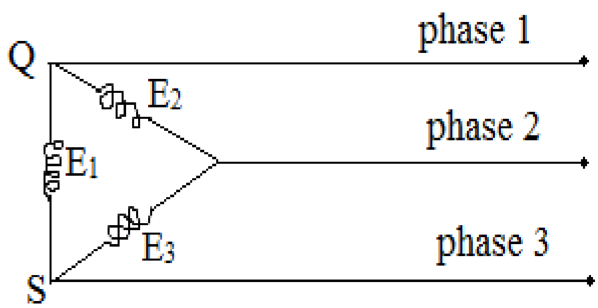

Fig.4.11

UNIT – V

Magnetic Properties of Materials

5.1: Magnetic Induction (B) :-

If a positive test charge of moving with velocity (V) through a point in a magnetic field experiences a force F, then the magnetic induction B at that point is defined by

$$f = qV \text{ x } B$$

$$\text{The magnitude of the Magnetic induction} = B = \frac{F}{qV\sin\theta}.$$

Here θ is the angle between V and B. The magnetic field can be represented by lines of induction. The tangent to the line of induction at any point gives the direction of B.

Unit : $Weber/m^2$ (or) $\frac{Newton}{Ampere - m}$ (or) $Tesla$

5.2: Magnetisation (M) :

Magnetisation M of the material is defined as the magnetic dipole moment induced per unit volume of the material.

Let 'm' be the magnetic dipole moment of a specimen of volume V. Then

$$M = \frac{m}{V}$$

In unmagnetized matter M will be zero.

Unit : $Ampere/metre\,(Am^{-1})$.

5.3: Magnetic flux :

The number of magnetic induction lines cutting through the surface is called magnetic flux.

$$i.e., \theta = B/S$$

Where S is the surface area

Unit : $weber$.

5.4 Relation between the three magnetic vectors B, H and M

Consider a Rowland ring having a toroidal winding of N turns around it. When a current i_o is sent through the winding, the ring is magnetized along it circumferential length. Fig.5.1 shows a section of magnetised ring. The small circles represent the current loops. The magnetisation arises due to the alignment of these current loops.

No net current inside the current loops because the adjacent currents are in the opposite direction. The currents in the outer portions of the outer – most loops remain uncancelled. Therefore the numerous inside current loops can be replaced by a single closed current i_s. Such a current is called *Amperian current***.**

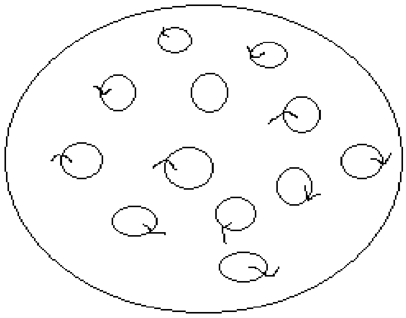

Fig.5.1

Let A = Area of cross section

l = Circumferential length of the ring.

Then volume $V = lA$

The ring behaves like a large dipole of magnetic moment.

$$m = i_s A$$

$$\therefore \text{Magnetization } M = \frac{m}{V} = \frac{i_s A}{lA} = \frac{i_s}{l}$$

The magnetization, therefore, is the surface current per unit length of the ring. This is commonly called *magnetization current***.**

Now the magnetic induction (B) within the material arises due to the free current i_o and due to the magnetisation of the ring it self $i.e., \frac{i_s}{l}$

$$\therefore B = \mu_0 \left(\frac{Ni_o}{l} + \frac{i_s}{l} \right) = \mu_0 \left(\frac{Ni_o}{l} + M \right)$$

or $$\frac{B}{\mu_0} - M = \frac{Ni_o}{l}$$

The quantity $\frac{B}{\mu_0} - M$ is called magnetizing field or magnetic field intensity H, ie.,

$$\frac{B}{\mu_0} - M = H$$

(or) $$B = \mu_0 (H + M)$$

In vector form $$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

This is the relation between the three vector B, H and M.

5.5: Magnetic susceptibility (χ_m) :-

Experimentally found that, in para and dia magnetic materials, the magnetisation M is proportional to the magnetic field intensity H. That is,

$$M \ \alpha \ H \quad (or) \ M = \chi_m H$$

The constant χ_m is called the magnetic susceptibility of the materials.

It may be defined as the ratio of the magnetization M to the magnetic field intensity H.

$$i.e., \chi_m = \frac{M}{H}$$

$\therefore$ the magnetic susceptibility of a material is defined as the intensity of magnetization acquired by the material per unit field strength.

We can classify magnetic material interms of susceptibility (χ_m).

If χ_m is +ve but small, the material is paramagnetic.

If χ_m is +ve but large, the material is Ferromagnetic.

If χ_m is +ve, the material is diamagnetic.

5.6: Magnetic Permeability (μ) :-

Consider the relation

$$\begin{aligned} B &= \mu_0 (H + M) \\ &= \mu_0 (H + \chi_m H) \\ &= \mu_0 (1 + \chi_m) H \\ &= \mu H \end{aligned}$$

Where $\mu = \mu_0 (1 + \chi_m)$ is called the magnetic permeability of the material.

$$\therefore B = \mu H$$

Magnetic permeability (μ) of a medium is defined the ratio of magnetic induction to the intensity of magnetic field.

$$\mu = \frac{B}{H}$$

For vacuum $\chi_m = 0$ and $\mu = \mu_0$

Hence magnetic induction in vacuum is $B_0 = \mu_0 H$

The ratio $\frac{B}{B_0} = \frac{\mu}{\mu_0} = \mu_r$

Called the relative permeability (μ_r). Obviously

$$\mu_r = 1 + \chi_m .$$

We may also classify magnetic materials in terms of the relative permeability μ_r .

Diamagnetism	:	$\mu_r < 1$
Paramagnetism	:	$\mu_r > 1$
Ferromagnetism	:	$\mu_r >> 1$

5.7: The Electron theory of Magnetism

The Paramagnetic, diamagnetic and Ferromagnetic behaviour of substances can be explained in terms of electron theory of matter.

Each electron is resolving around the nucleus. Each moving electrons behaves like a tiny current loop and therefore possesses a *orbital magnetic dipole moment*. Furthermore, each electron is spinning about an axis through itself. This spin also gives rise to a magnetic dipole moment called *spin magnetic dipole moment*. In general, the resultant magnetic dipole moment of an atom is the vector sum of the orbital and spin magnetic dipole moments of its electrons.

(i) Explanation of Diamagnetism :-

Diamagnetism occurs when atom consists of an even number of electrons. The electrons of such atoms are paired. The electrons in each pair have orbital motions as well as spin motions in opposite sense. The resultant magnetic dipole moment of the atom is then zero. Hence when such a substance is placed in a magnetic field, the field does not tend to align the dipoles of the substance. However, the field modifies the motion of the electrons in orbits which are equivalent to tiny current loops. The electron pair and hence the atom, thus acquire an effective magnetic dipole moment which is opposite to the applied field. Hence for diamagnetic materials μ is opposite to H. So the susceptibility χ_m of a diamagnetic substance is negative and is very small.

(ii) Explanation of Paramagnetism

In paramagnetic materials, the magnetic field associated with the orbiting and spinning electrons do not cancel out. There is a net intrinsic moment in it. The molecules in it behaves like little magnets. When such a substance is placed in an external magnetic field, it will turn and line up with its axis parallel of the external field. Since μ and H are in the same direction in paramagnetics, the susceptibility χ_m is +ve. The magnetization of paramagnetic substances decreases at the temperature of the substance increases, ie $\chi_m \propto \frac{1}{T}$

(iii) Explanation of Ferromagnetism

Ferromagnetic substances are strongly magnetic. A ferromagnetic has a spontaneous magnetic moment – a magnetic moment even in zero applied field. The atoms or molecules of ferromagnetic materials have a net intrinsic magnetic dipole moment which is primarily due to the spin of the electrons. The interaction between the neighbouring magnetic dipoles is very strong. It is called *exchange interaction* and it is present even in the absence of an external magnetic field.

This effect of the exchange interaction to align the neighbouring magnetic dipole moment parallel one another spreads over a small finite volume bulk. This small volume of the bulk is called a *domain* (Fig .5.2). All magnetic moments within a domain will point in the same direction resulting in a large magnetic moment.

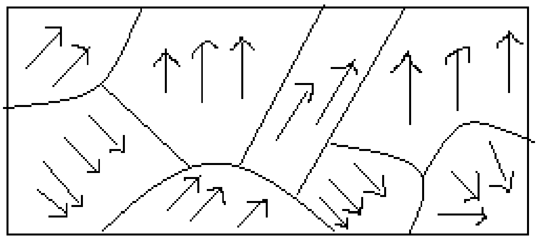

Fig.5.2

Thus the bulk material consists of many domains. The domains are oriented in different directions. The total magnetic moment of a sample of the substance is the vector sum of the magnetic moments of the component domain.

At very high temperatures, the ferromagnetic materials become paramagnetic materials. The critical temperature above which a ferromagnetic materials become a paramagnetic material is called *the curie temperature.*

5.8: Determination of Susceptibility – Curie balance method

The Curie balance method to find the susceptibility of the specimen is shown in Fig.5.3. The specimen is kept inside a porcelain bulb B. The Porcelain bulb is attached to one end of a long fibre F form a torsion head T. The other end of the arm carries a scale pan P into which suitable weights can be put to obtain balance. A damper D is used to prevent the disturbing oscillations. A Pointer P_1 moving on a calibrated. scale S measures the displacement produced.

The magnetic field is supplied by the pole pieces N and S of an electromagnet which are kept inclined at an angle of $70°$ with respect to the axis of symmetry. When the magnetic

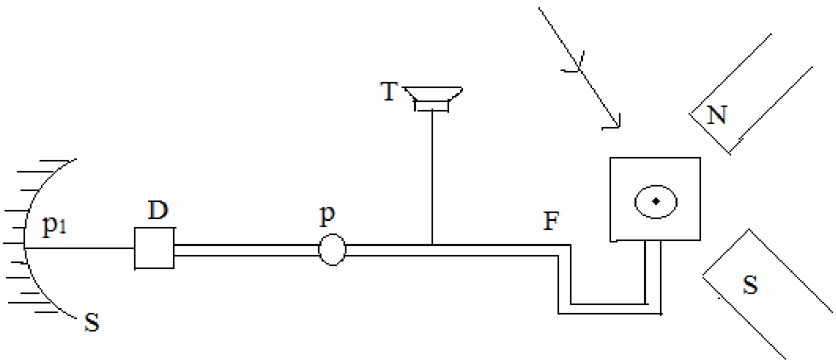

Fig.5.3

field is switched on, the points P_1 gets displaced. The displacement can be measure correct to 0.001mm using a sensitive microscope. Measuring the displacement and knowing the elastic constants of suspension fibre, the force acting on the specimen can be estimated.

If the specimen has a susceptibility χ_1 and is immersed in a medium with susceptibility χ_2. The force

$$F_x = (\chi_1 - \chi_2)\ \mu_0\ VH_y \frac{dH_y}{dx}$$

When F_x is the force along $x-$axis , V the volume of the specimen, H the intensity of magnetic field perpendicular to $x-$axis , dH_y/dX is the rate of change of magnetic field.

The value of H_y is determined from the search coil and Ballistic galvanometer. The value of dH_y/dx is found from the graph of H_y against x. Knowing F_x and V the susceptibility χ_1 of the specimen can be estimated at any given temperature.

Curie's method is also suitable for the measures of the magnetic susceptibilities of paramagnetic liquid and gases.

5.9: Moving Coil Ballistic Galvanometer

Principle :-

When a current is passed through a coil, suspended freely in a magnetic field, it experiences a force in a direction given by Fleming's left hand rule.

(i) Construction :

It consists of a rectangular coil of thin copper wire wound on a non – metallic frame of ivory (Fig.5.4). It is suspended by means of a phosphor – bronze wire between the poles of a powerful horse – shoe magnet. A small circular mirror is attached to the suspension wire. Lower end of the coil is connected to a hair – spring. The upper end of the suspension wire and the lower end of the spring are connected to terminals T_1 *and* T_2. A cylindrical soft iron core (C) is placed symmetrically inside the coil between the magnetic poles. This iron core concentrates the magnetic field and helps in producing radial field.

Ordinary Galvanometer is used to measure current. But B.G is used to measure electric charge.

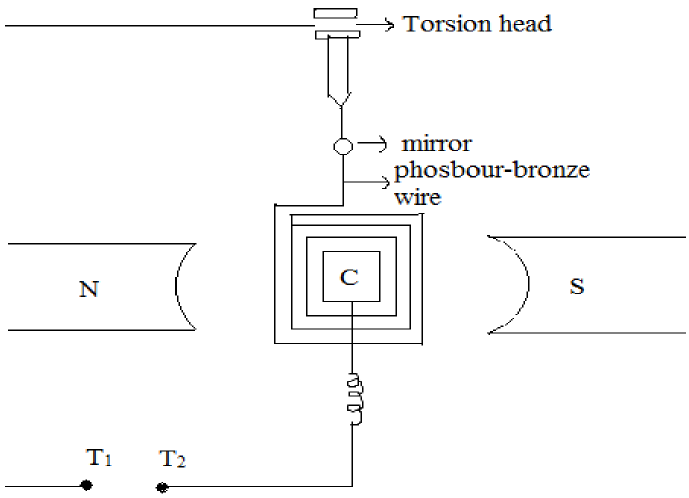

Fig.5.4

(ii). Theory :-

(i) Consider a rectangular coil of N turns place in uniform magnetic field of magnetic induction B (Fig.5.5) 'l' be the length of the coil and 'b' its breadth.

Area of the coil $A = lb$

When a current i passing through the coil, then torque on the coil

$$\tau = N i B A \tag{5.1}$$

If the current passes for a short interval dt, the angular impulse produced in the coil is

$$\tau\ dt = N i B A dt \tag{5.2}$$

If the current passes for t secs, the total angular impulse given to the coil is

$$\int_0^t \tau\ dt = \int_0^t N B A i\, dt = N B A \int_0^t idt \tag{5.3}$$

Where $q = \int_0^t idt$ = total charge passing through the galvanometer.

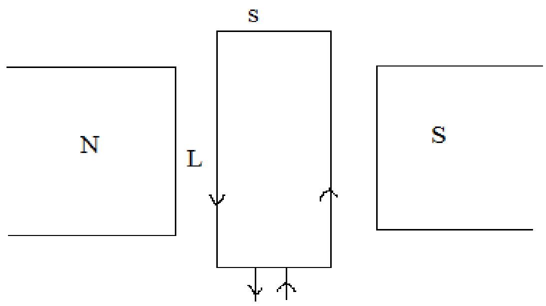

Fig.5.5

Let I be the moment of inertia of the coil and $'\omega'$ to its angular velocity, then change in angular momention of the coil is $I\omega$

$$\therefore \quad I\omega = NBAq \qquad (5.4)$$

ii) The kinetic energy of the moving system $\frac{1}{2} I\omega^2$ is used in twisting the suspension wire through an angle θ. Let C be the restoring torque per unit twist of the suspension wire. Then,

$$\text{Work done} = \frac{1}{2} C\theta^2$$

$$\therefore \frac{1}{2} I\omega^2 = \frac{1}{2} C\theta^2$$

$$\text{(or)} \quad I\omega^2 = C\theta^2 \qquad (5.5)$$

(ii) The period of oscillation of the coil is

$$T = 2\pi\sqrt{I/C} \quad (or) \quad T^2 = \frac{4\pi^2 I}{C}$$

$$\therefore I = \frac{T^2 C}{4\pi^2} \qquad (5.6)$$

Multiplying eqs. (5.5) and (5.6), $I^2\,\omega^2 = \dfrac{C^2 T^2 \theta^2}{4\pi^2}$

$$\text{(or)} \quad I\,\omega = \frac{CT\theta}{2\pi} \qquad (5.7)$$

Equating (5.4) and (5.7), $NBAq = \dfrac{CT\theta}{2\pi}$

(or)

$$q = \left(\frac{T}{2\pi}\right)\left(\frac{C}{NBA}\right)\theta \qquad (5.8)$$

This gives the relation between the charge flowing and the ballistic throw θ of the galvanometer, $q \propto \theta$

$\left(\frac{T}{2\pi}\right)\left(\frac{C}{NBA}\right)$ is called the ballistic reduction factor (k)

$$q = k\,\theta \tag{5.9}$$

(iii) Correction for damping in Ballistic Galvanometer

In Eq.(5.9), the correct value of first throw is obtained by applying damping correction, Let $\theta_1, \theta_2, \theta_3 ...$ be the successive maximum deflection from zero position to the right and left (Fig.5.6).

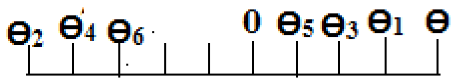

Fig.5.6

Then it is found that

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} .... = d \tag{5.10}$$

The constant d is called the decrement per half vibration.

Let $d = e^{\lambda}$ so that $\lambda = \log_e d$

Let θ be the true first throw in the absence of damping.

$\theta > \theta_1$. The first throw θ_1, is observed after the coil completes a quarter of vibration.

In this case, the value of the decrement would be $e^{\lambda/2}$.

$$\therefore \frac{\theta}{\theta_1} = e^{\lambda/2} = \left(1 + \frac{\lambda}{2}\right)$$

(or) $$\theta = \theta_1\left(1 + \frac{\lambda}{2}\right)$$

We can calculate λ by observing the first throw θ_1 and the eleventh throw θ_{11}.

$$\frac{\theta_1}{\theta_{11}} = \frac{\theta_1}{\theta_2} \text{ x } \frac{\theta_2}{\theta_3} \text{ x } \frac{\theta_3}{\theta_4} \text{ x } \frac{\theta_4}{\theta 5} \text{ x } \frac{\theta_5}{\theta_6} \text{ x } \frac{\theta_6}{\theta_7} \text{ x } \frac{\theta_7}{\theta_8} \text{ x } \frac{\theta_8}{\theta_9} \text{ x } \frac{\theta_9}{\theta_{10}} \text{ x } \frac{\theta_{10}}{\theta_{11}}$$

$$\frac{\theta_1}{\theta_{11}} = e^{10\lambda} \left(\because \frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} \text{ x } \frac{\theta_2}{\theta_3} = \text{d}^2 = e^{2\lambda} \right)$$

(or) $$\lambda = \frac{1}{10}\log_e \frac{\theta_1}{\theta_{11}}$$

$$\frac{2.3026}{10}\log_{10} \frac{\theta_1}{\theta_{11}} \qquad (5.12)$$

Where λ is called the *logarithmic decrement***.**

$$\therefore q = \left(\frac{T}{2\pi}\right)\left(\frac{C}{NBA}\right)\theta_1\left(1+\frac{\lambda}{2}\right) \qquad (5.13)$$

(iv) Dead beat and Ballistic Galanometers :

Galvanometers are classified as (i) dead beat (or) a periodic and (ii) ballistic galvanometers.

A moving coil galvanometer in which the coil is wound on a metallic conducting frame is known as a dead beat galvanometer. It is called **"***dead – beat***"** because it gives a steady deflection without producing any oscillation when a steady current is passed through the coil.

(v) Conditions for a moving coil galvanometer to be dead beat.

(i) Moment of inertia of the system should be small.

(ii) Coil should be mounted on a conducting frame.

(iii) Suspension fibre should be comparatively thicker.

(vi) Conditions for a moving coil galvanometer to be ballistic.

(i) The moment of inertia of moving system should be large.

(ii) Air resistance should be small.

(iii) Suspension fibre should be very fine.

(iv) The damping should be small, ie the coil should be wound on a non – conducting frame.

(vii) Current and voltage sensitive of a moving – coil galvanometer :-

The figure of merit or current sensitivity (S_C) of a moving coil mirror galvanometer is the current that is required to produce a deflection of 1mm on a scale kept at a distance of 1 metre from the mirror.

It is expressed in $\mu A/mm$.

The voltage sensitivity (S_V) is the potential difference (p.d) that should be applied to the galvanometer to produce a deflection of 1mm on a scale at a distance of 1 metre.

It is expressed in $\mu V/mm$.

5.10 Measurement of charge sensitiveness (Figure of merit a B.G)

The charge passing through a B.G is given by

$$q = \frac{T}{2\pi}\frac{C}{NBA}\theta_1\left(1+\frac{\lambda}{2}\right) = k\theta_1\left(1+\frac{\lambda}{2}\right)$$

Where k is charge sensitiveness or figure of merit of the galvanometer. It is also known as the "ballistic reduction factor" of the galvanometer.

Two resistance boxes P and Q and a key k are connected in series with an accumulator of *emf* E (Fig.5.7). A capacitor of known capacitance C is connected to P through the vibrator V and charging terminal *ch* of the charge – discharge key. The capacitor is charged with the p.d across P . The charge on the capacitor can be discharged through the *B.G.* included in the circuit through the vibrator and discharge terminal of the charge – discharge key. A commutator C_r is included in the circuit to reverse the charge in the *B.G.*

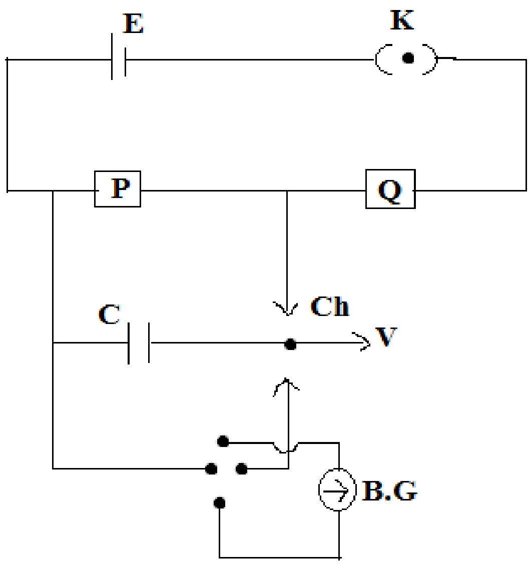

Fig.5.7

1000Ω in P and 9000Ω in Q are included. The capacitor is charged and immediately discharged through the *B.G.* The first throw θ_1 is noted. The experiment is repeated with $P = 2000\Omega$, 3000Ω etc., keeping $P+Q = 10{,}000\Omega$. Mean value of P/θ_1 in calculated.

Let the capacitance of the capacitor be $C\ \mu F$

Charge on the capacitor $q = \frac{EP}{P+Q} \text{ x } C\ \mu C$

This charge produces a throw θ_1.

Undamped throw $\theta = \theta_1 (1 + \frac{1}{2}\lambda)$

Charge required to produce unit deflection = k

$$\therefore k\theta_1 (1 + \frac{1}{2}\lambda) = \frac{EP}{(P+Q)} \text{ x } C$$

or

$$k \quad \frac{EC}{(P+Q)} \text{ x } \frac{P}{\theta_1 \text{ x } (1 + \frac{1}{2}\lambda)} \mu C / div$$

The value of λ is obtained by observing the first throw θ_1 and then eleventh throw θ_{11} and using the relation

$$\lambda = \frac{1}{10} \log e \frac{\theta_1}{\theta_{11}}$$

$$= \frac{1}{10} \text{ x } 2.3026 \text{ x } \log_{10} \frac{\theta_1}{\theta_{11}}$$

5.11: Uses of Ballistic galvanometer

Absolute Capacitance of a Capacitor :-

(i) Two resistance boxes P and Q are connected in series with an accumulator of *emf* E (Fig.5.8). A small resistance $(= 0.1\Omega)$ is taken in P and a large resistance (9999.9Ω) in Ω so that $P + Q = 10{,}000\Omega$. The mirror galvanometer (MG) and a resistance box R are connected across P. With no resistance in R, the steady deflection d of the galvanometer is found. A suitable resistance is taken in R till the deflection becomes half. The resistance in R is the galvanometer resistance Rg. The experiment is repeated for various values of P keeping P+Q constant.

Current through galvanometer $= \frac{EP}{P+Q} \text{ x } \frac{1}{R_g}$ (5.14)

Current through the galvanometer is also $= \frac{C}{BAN} d$ (5.15)

From Eqs (5.14) and (5.15)

$$\frac{C}{BAN} d = \frac{EP}{P+Q} \text{ x } \frac{1}{R_g}$$

$$\therefore \frac{C}{BAN} = \frac{E}{(P+Q)} \text{x} \left(\frac{p}{d}\right) \text{x} \frac{1}{R_g} \qquad (5.16)$$

The mean value of p/d is found out from this expt.

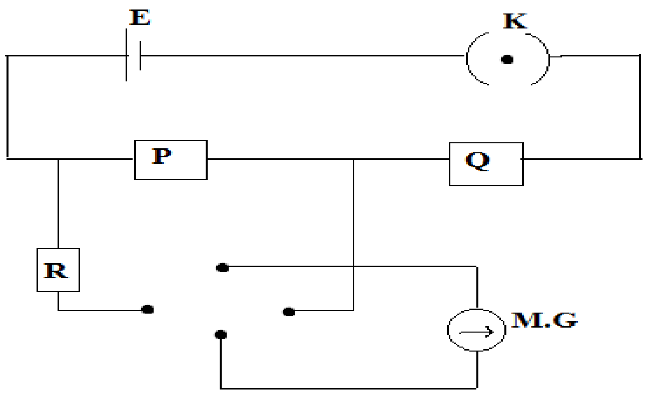

Fig.5.8

(ii) the galvanometer coil is set oscillating freely in open circuit. The time for 10 oscillations is found and the period T is calculated.

(iii) Connections are made as shown in Fig.5.9. Resistances $P_1 (1000\Omega)$ and $Q_1 (9000\Omega)$ are included in the boxes P and Q respectively.

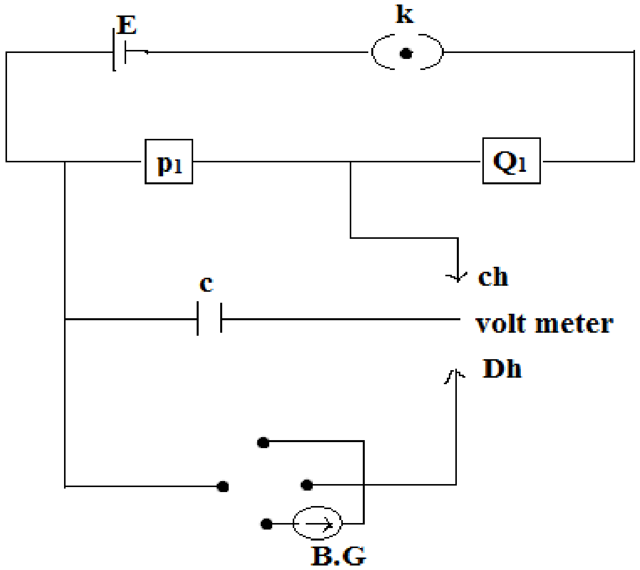

Fig.5.9

Potential difference across P_1

$$V = \frac{EP_1}{P_1 + Q_1}$$

The drop of potential across P_1 is used to charge the capacitor by connecting the terminals ch and V of the charge discharge key.

Charge on the capacitor = $q = CV = C \times \frac{EP_1}{P_1 + Q_1}$ (5.17)

The terminals Dh and V are now connected so that the capacitor gets discharged through the galvanometer. The first throw θ_1 is noted.

$$q = \frac{T}{2\pi} \frac{C}{NBA} \theta_1 \left(1 + \frac{1}{2}\lambda\right) \quad (5.18)$$

$$C \times \frac{EP_1}{P_1 + Q_1} = \frac{T}{2\pi} \frac{C}{NBA} \theta_1 \left(1 + \frac{1}{2}\lambda\right)$$

and from Eqs (5.17) and (5.18)

or $$C = \frac{T}{2\pi} \frac{C}{NBA} \left(\frac{\theta_1}{P_1}\right) \frac{P_1 + Q_1}{E} \left(1 + \frac{1}{2}\lambda\right) \quad (5.19)$$

Substituting the value of $\left(C/_{BAN}\right)$ from Eq. (5.16) in Eq. (5.19).

$$C = \frac{T}{2\pi} \frac{E}{(P+Q)} \left(\frac{p}{d}\right) \frac{1}{Rg} \left(\frac{\theta_1}{P_1}\right) \left(\frac{P_1 + Q_1}{E}\right) \left(1 + \frac{1}{2}\lambda\right)$$

But $P + Q = P_1 + Q_1$

$$\therefore C = \frac{T}{2\pi} \frac{1}{Rg} \left(\frac{p}{d}\right) \left(\frac{\theta_1}{P_1}\right) \left(1 + \frac{1}{2}\lambda\right) \quad (5.20)$$

The experiment is repeated for various values of P_1, keeping $(P_1 + Q_1)$ same as $P + Q$. The mean value of $\theta_1/_{P_1}$ is calculated.

iv) To find λ, the coil is set oscillating. The first throw θ_1 and the eleventh thrown θ_{11} are noted. Then,

$$\lambda = \frac{2.3026}{10} \log_{10} \frac{\theta_1}{\theta_{11}}$$

Substituting the values of T, Rg, $T, Rg, (P/_d), \left(\theta_1/_{P_1}\right)$ and λ in Eq.(5.20), C is determined.